Persuasion of Loss-Averse Receivers Through Early Offers

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Persuasion of Loss-Averse Receivers Through Early Offers*

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Abstract

We study a simple bargaining model in which the sender can make an early offer to the receiver. Initially, the sender has private information about the value of the receiver's outside option. The receiver learns this value before she chooses between the sender's early offer and her outside option. Nevertheless, if the receiver is expectation-based loss averse, the sender can persuade her to accept an offer that is inferior to her outside option. This result is due to the interaction of two effects: the *attachment effect* that makes it costly for the receiver to reject an offer that she planned to accept, and the *uncertainty effect* which renders the acceptance of the sender's offer as the preferred plan since it creates peace of mind at an early stage. If the receiver faces uncertainty in multiple dimensions, the main result holds for all degrees of loss aversion. Thus, expectation-based loss-averse preferences imply that there is scope for persuasion through signaling even if the receiver has all payoff-relevant information at the decision stage.

Keywords: Reference-Dependent Preferences, Signaling, Loss Aversion, Early Offers
JEL Classification: D21, D83, L41

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1 Introduction

In many settings, a party with superior information tries to influence the choices of a less well-informed decision maker. Economists have extensively studied the scope for *persuasion* of rational decision makers through a variety of mechanisms: signaling (Spence 1973), cheap talk (Crawford and Sobel 1982), and Bayesian persuasion (Kamenica and Gentzkow 2011). A common feature of these mechanisms is that, at the point in time when the decision maker chooses an option, she has incomplete information about the state of world (such as the opponent's type or the realization of a payoff-relevant state variable). She only can infer details about the state of the world based on the actions of or the information provided by the informed party. In an environment with complete information at the decision stage, however, there is no scope for persuasion of rational decision makers with standard preferences.

This changes when the decision maker (she) is rational, but exhibits reference-dependent preferences. The actions of the informed party (he) may endogenously change the decision maker's reference point, which in turn affects her preferences over options at the decision stage. Hence, the informed party may be able to influence the choices of the decision maker with reference-dependent preferences, even if she has complete information about the state of the world at the point in time when she makes a final decision. This type of persuasion does not require any congruence of preferences (as in cheap talk) or commitment on the side of the sender (as in Bayesian persuasion).

In this paper, we study persuasion through signaling to receivers with reference-dependent preferences. To this end, we consider a simple dynamic model of bargaining between a sender and a receiver. The receiver initially does not know the value of her outside option. She learns this value before she makes a choice. The sender knows the receiver's outside option value right away. Before the receiver learns about her outside option, the sender can make a binding "early offer" to the receiver, i.e., an offer that is valid also in the last stage of the game when the receiver has complete information. The receiver then chooses between this early offer and her outside option. This framework captures different strategic situations.

Shopping Example. The sender is a seller and the receiver is a consumer shopping for a product. The seller has a competitive advantage relative to his rivals: He can profitably sell his product at a lower price (or offer higher quality). He also knows the configuration of the "standard product" which is sold by his rivals on a competitive market. In contrast, when observing the seller's offer, the consumer does not yet know her utility from the standard product. However, she learns this value at a later stage before choosing between the seller's offer and the standard product.

Negotiation Example. The sender and the receiver negotiate about an object. An agreement between the two parties would generate positive gains from trade. The sender is a seasoned negotiator who knows both his and the receiver's bargaining position. In contrast, the receiver is inexperienced and does not yet know her "BATNA" (the best alternative to a negotiated agreement) when observing the early offer.[1] Before making a final decision, the receiver learns her BATNA and decides about whether to accept or reject the sender's offer.

To capture the receiver's reference-dependent preferences, we assume that she is expectation-based loss averse (Kőszegi and Rabin 2006, 2007). After observing the receiver's early offer, she updates her beliefs about the value of the outside option. Next, she makes a plan under what circumstances she accepts which option. Her beliefs and plan jointly determine her reference point. This reference point defines her preferences at the decision stage.

Under standard preferences, the sender would have to make an offer that is at least as good for the receiver as the outside option, otherwise the receiver would reject it. We show that, if the receiver is loss-averse, then an equilibrium may exist in which the sender persuades the receiver to accept an early offer that has a lower total value than her outside option (henceforth, an "inferior option"). For this, the sender needs to differentiate his offer from the outside option, i.e., it needs to have a feature that the outside option does not have. We allow the total value of the early offer to consist of a regular value and a transfer. A loss-averse receiver treats the regular value and the transfer dimension separately.[2] The regular value occurs in the same dimension as the outside option value, so that these are directly comparable for the receiver. The transfer occurs in a dimension in which the outside option only offers a zero outcome. Therefore, if the receiver is loss-averse, the sender can differentiate his offer from the outside option through the transfer without changing its total value.

As an illustration, suppose the early offer has a regular value below the outside option value and a positive transfer. For example, the seller makes an offer that is more expensive than the standard product, but offers on-site repair services, a convenience that his (online) rivals cannot provide. In this case, if the receiver's reference point is defined by the plan "accept the early offer", then choosing the outside option creates a gain in the regular value dimension and a loss in the transfer dimension. Loss aversion (the tendency that losses loom larger than gains of similar size) then reduces the payoff from accepting the outside option. This enables the sender to redistribute surplus from the receiver to himself.

Whether such redistribution takes place in equilibrium depends on how the sender makes

[1] The BATNA concept is motivated, for example, by Fisher et al. (2011).

[2] This separation is a feature of the expectations-based reference point model of Kőszegi and Rabin (2006, 2007) and is closely linked to mental accounting and the endowment effect. It describes individuals' tendency to assess gains and losses separately across different dimensions (Kahneman et al. 1990, 1991, Thaler 1985, 1999).

early offers to the receiver. Note that the receiver could just plan to reject the sender's offer and accept the outside option with certainty. Two different effects – induced by expectation-based loss-averse preferences – interact to enable persuasion through signaling: the *attachment effect* and the *uncertainty effect*. The *attachment effect* implies that it is costly for the receiver to choose the outside option, provided that accepting the early offer determines the reference point. As described above, this effect is caused by gain-loss sensations in the regular value and transfer dimension. The *uncertainty effect* makes the acceptance of the early offer relatively more attractive than the acceptance of the outside option. Planing the acceptance of the sender's offer creates peace of mind at an early stage, while planing the acceptance of the outside option exposes the receiver to uncertainty, which in turn lowers her expected payoff. Both the attachment and the uncertainty effect must be strong enough so that it is optimal for the receiver to plan the acceptance of and eventually accept an offer that with certainty is less valuable for her than the outside option. An equilibrium that is optimal for the sender balances the relative strength of these two effects.

We examine the structure of equilibria in which, for any realization of the outside option value, the sender persuades the receiver to accept an offer that is inferior to her outside option. When the receiver faces uncertainty only in one outcome dimension, such an equilibrium exits if the loss aversion parameters – the weight of gain-loss sensations η and the degree of loss aversion λ – are sufficiently large. Such an equilibrium must be a semi-separating signaling equilibrium. To generate an uncertainty effect, the sender's early offer informs the receiver about the range of possible outside option values, but not the precise number. Thus, it has an interval structure, somewhat reminiscent of an equilibrium with information transmission in a cheap talk game (Crawford and Sobel 1982). We show that any sender-preferred equilibrium exhibits this feature if there is uncertainty only in one outcome dimension.

In our baseline model, persuasion through signaling is possible only if the loss aversion parameters are large. Specifically, we need that $\eta(\lambda - 1) > 3$. As we discuss in the next section, these are empirically relevant degrees of loss aversion as there is substantial evidence for the uncertainty effect. Nevertheless, in most theoretical applications, the assumed levels of loss aversion are typically smaller. In some applications, large levels of loss aversion are ruled out explicitly (e.g., Herweg and Mierendorff 2013).

Therefore, we show that our main result holds for all loss aversion parameter values η, λ that satisfy $\eta(\lambda - 1) > 0$ if the receiver faces uncertainty about the outside option in multiple outcome dimensions. For example, she may be uncertain about details of the product specification of the outside option, such as design, customer service, delivery times, or warranties. In contrast, she observes all these details for the sender's product through the early offer. Uncertainty in multiple dimensions implies that the plan "accept the outside option" exposes the receiver to gain-loss sensations even if the sender's offer would perfectly signal the total value of the outside option to the receiver at an early stage. It turns out that this feature of the environment allows the sender to persuade the receiver to accept an inferior offer, regardless of the degree of loss aversion. Thus, expectation-based loss-averse preferences imply that there is scope for persuasion through signaling even if the receiver has all payoff-relevant information at the decision stage. Importantly, we show that the sender still may benefit from having an interval structure in the signaling equilibrium, i.e., from making offers so that the receiver cannot infer the precise value of her outside option, but only an interval of potential outcomes.

The rest of the paper is organized as follows. In Section 2, we explain how our paper contributes to the related literature on expectation-based loss-averse preferences, the uncertainty effect, and persuasion. In Section 3, we introduce the formal model and define the equilibrium concept. In Section 4, we derive our main result for the baseline model and characterize the sender-preferred equilibrium. In Section 5, we consider uncertainty in multiple dimension and show that in such a setting our main result obtains for any positive degree of loss aversion. In Section 6, we examine a number of extensions and robustness checks of our baseline model. Section 7 concludes. All proofs are relegated to the appendix.

2 Related Literature

Strategic interaction of agents with expectation-based loss-averse preferences. Our paper mainly contributes to the literature that analyzes the implications of expectation-based loss aversion for strategic interaction. The most closely related papers in this literature study a monopolist's optimal pricing and marketing strategy when consumers are expectations-based loss averse. Herweg and Mierendorff (2013) consider a setting in which consumers face demand uncertainty. Loss-averse consumers may strictly prefer a flat rate tariff to a measured tariff even if it is not the option that minimizes their expected expenses. This makes it optimal for the monopolist to offer flat rate contracts. Heidhues and Kőszegi (2014) and Rosato (2016) show that a monopolist can exploit the consumers' loss aversion by creating attachment to its product through commitment to sophisticated price strategies: Heidhues and Kőszegi (2014) consider mixtures of sales and regular prices, Rosato (2016) quantity restrictions on products that are on sale. Karle and Schumacher (2017) demonstrate that a monopolist can also use the partial revelation of match value information to create consumer attachment. Further, Hancart (2023) shows that the monopolist may randomize over prices in equilibrium even if it cannot commit to its strategy (as in Heidhues and Kőszegi 2014).[3] Importantly, the results in these papers only rely on the attachment effect: A loss-averse consumer who expects to obtain a product will find it more difficult to decide against its purchase than a consumer who expects not to own it. The contribution of the present paper in this literature is to consider the interaction of the attachment and the uncertainty effect.

Uncertainty Effect. Several experimental studies find versions of the uncertainty effect. Gneezy et al. (2006) first demonstrated that some individuals value a lottery less than its worst outcome. They applied a between-subject design and obtained the same result for different types of goods, elicitation methods, and implementation. Sonsino (2008) finds in auctions for single gifts and binary lotteries on these gifts that 27 percent of subjects sometimes submit higher bids for the single gift than for the lottery even though the lottery's worst outcome is the gift. In a post-experimental survey, many participants indicate "aversion to lotteries" as their explanation for such behavior. Simonsohn (2009) conducts several within-subject variations of the experiment by Gneezy et al. (2006) and finds that 62 percent of subjects exhibit the uncertainty effect. Newman and Mochon (2012) demonstrate that these results also hold in settings that largely avoid disappointments, i.e., there are different potential outcomes, but they are all valued roughly the same. Yang et al. (2013) show that a pronounced uncertainty effect occurs if the certain outcome is framed as a "gift certificate" while the lottery is framed as "lottery ticket" (or coin flip, gamble, raffle). Mislavsky and Simonsohn (2018) find the uncertainty effect when subjects perceive the certain outcome as more natural transaction than the lottery. They interpret the lottery as a transaction that has an unexplained feature.[4]

Despite this evidence, only few papers have examined the uncertainty effect in strategic settings so far. Dreyfuss et al. (2022) and Meisner and von Wangenheim (2023) explore whether expectation-based reference-dependent preferences can explain misrepresentations in deferred acceptance mechanisms (which are known to be strategy-proof). Both in experiments and in the field, there is a substantial share of individuals who chooses first-order stochastically dominated options.[5] Loss-averse individuals may employ such behavior in order to avoid disappointments. In the context of product switching, our companion paper (Karle et al.

[3]Further applications of expectation-based loss aversion include Heidhues and Kőszegi (2008), Karle and Peitz (2014), and Karle and Möller (2020) on imperfect competition; Rosato (2017) on sequential bargaining; Benkert (2022) on bilateral trade; Carbajal and Ely (2016) and Hahn et al. (2018) on monopolistic screening; Herweg et al. (2010) and Macera (2018) on principal-agent contracts; Lange and Ratan (2010), Dato et al. (2018), and Balzer and Rosato (2021) on auctions or tournaments; Dato et al. (2017) on strategic interaction in finite games; and Daido and Murooka (2016) on team incentives.

[4]In addition, Andreoni and Sprenger (2011) also find the uncertainty effect in their experimental data. Some studies demonstrate that the uncertainty effect does not show up under certain conditions; see Rydval et al. (2009) and Wang et al. (2013).

[5]For example, they may rank a funded position (study place with a fellowship) below an identical nonfunded one; see Dreyfuss et al. (2022) for a summary of the empirical evidence.

2023) motivates the idea that the uncertainty effect can generate scale-dependent psychological switching costs. The present paper is the first that examines the interaction of attachment and uncertainty effect in a strategic setting.

Persuasion with behavioral receivers. More generally, we contribute to the literature that considers persuasion with boundedly rational senders or receivers, see, for example, Hagenbach and Koessler (2017) as well as Bilancini and Boncinelli (2018) for signaling, and Blume and Board (2013), Glazer and Rubinstein (2012, 2014), Galperti (2019), Giovannoni and Xiong (2019), Hagenbach and Koessler (2020), as well as Eliaz et al. (2021) for cheap talk. In contrast to these papers, we assume that agents have fully rational beliefs, while the receiver exhibits expectation-based loss-averse preferences. The main innovation of our model is that it allows for persuasion through signaling in a setting where the receiver is perfectly informed about the realization of all payoff-relevant variables when she chooses between options.

3 The Model

A loss-neutral sender interacts with a loss-averse receiver in two periods. In period 1, the sender makes an early offer (v^s, t^s) to the receiver, where $v^s \in [0, 1]$ is the regular value of the offer to the receiver and $t^s \in \mathbb{R}_+$ a transfer from the sender to the receiver. In period 2, the receiver learns about her outside option $(v^o, 0)$, where $v^o \in [0, 1]$. She then chooses between the sender's offer (v^s, t^s) and her outside option $(v^o, 0)$. The distinction between regular value and transfer allows the sender to offer something that the outside option does not provide, e.g., through product differentiation.

If the receiver accepts the outside option, her consumption utility is v^o and the sender's payoff is zero. If the receiver accepts the sender's offer, her consumption utility is $v^s + t^s$ and the sender's payoff is $1 - v^s - t^s$. The shape of the sender's payoff function ensures that the sender can profitably trade with the receiver even if the receiver's outside option is maximal, and that regular value and transfer are fungible for the sender. The outside option value v^o is distributed according to the distribution function F with continuously differentiable density f. We assume that F has full support on the unit interval and that f is bounded, so that we have $0 < f(v^o) < \infty$ for all $v^o \in [0, 1]$. Moreover, for several results we will require that F is weakly convex. In period 1, the sender observes the realization of v^o and can condition his offer on this value, while the receiver only knows the distribution of v^o. Figure 1 shows the timeline of the interaction between sender and receiver.

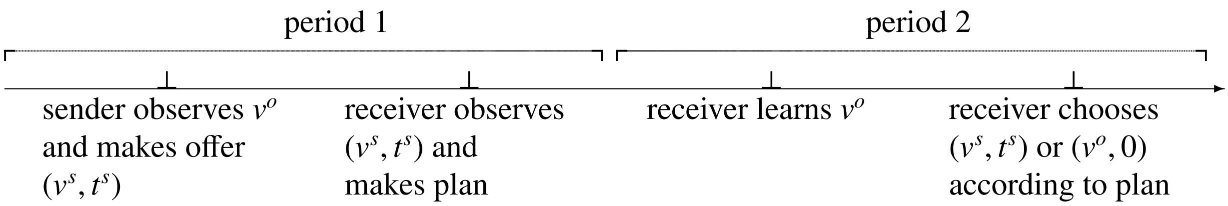

Figure 1: Timeline

Preferences. To model the receiver's expectation-based loss aversion we follow Kőszegi and Rabin (2006, 2007). Her payoff from accepting an option in period 2 consists of two components: consumption utility and gain-loss utility from comparisons of the actual outcome to a reference point. This reference point is defined by the receiver's period-1 expectations. Suppose that in period 1 she expects to accept the option $(\tilde{v}, \tilde{t})$ with certainty in period 2. If she indeed accepts option (v, t), her payoff equals

$$U(v, t \mid \tilde{v}, \tilde{t}) = v + t + \mu(v - \tilde{v}) + \mu(t - \tilde{t}). \tag{1}$$

The function μ captures gain-loss utility. We assume that μ is piecewise linear with slope η for gains and slope $\eta\lambda$ for losses; $\eta > 0$ is the weight of gain-loss utility relative to consumption utility, and $\lambda > 1$ is the receiver's degree of loss aversion.

The receiver may have stochastic expectations over outcomes. Let the distribution functions G^v and G^t denote her period-1 expectations regarding the outcome in the value and transfer dimension, respectively. The receiver's payoff from accepting option (v, t) is given by

$$U(v, t \mid G^v, G^t) = v + t + \int \mu(v - \tilde{v})\, \mathrm{d}G^v(\tilde{v}) + \int \mu(t - \tilde{t})\, \mathrm{d}G^t(\tilde{t}). \tag{2}$$

Thus, gains and losses are weighted by the probability with which the receiver expects them to occur. This preference model captures the following intuition. If the receiver expects to get either 0 or 1 in the value dimension, each with probability 50 percent, then an allocation of 0.6 feels like a gain of 0.6 weighted with 50 percent probability, and a loss of 0.4 also weighted with 50 percent probability.

Strategies and Equilibrium. The sender's strategy defines his offer (v^s, t^s) in period 1 based on the receiver's outside option value v^o. It is given by the measurable function[6]

$$\sigma^s : [0, 1] \rightarrow [0, 1] \times \mathbb{R}_+. \tag{3}$$

Thus, the sender's offer is potentially informative for the receiver about her outside option value v^o. Upon observing the sender's offer (v^s, t^s), the receiver updates her belief about her outside option value to $\hat{F} \equiv F(v^o \mid v^s, t^s)$. She then makes a plan under what circumstances she accepts either the outside option or the sender's offer. Formally, the receiver's plan is a strategy that defines her choice in period 2 based on the outside option and the sender's offer,

$$\sigma^r : [0, 1] \times [0, 1] \times \mathbb{R}_+ \rightarrow [0, 1] \times \mathbb{R}_+. \tag{4}$$

Given sender's strategy σ^s, his offer (v^s, t^s), and the receiver's strategy σ^r, we can define the receiver's expectations about period-2 outcomes. Let $G^v \equiv G^v(\tilde{v} \mid \sigma^s, \sigma^r, (v^s, t^s))$ denote her expectations about the outcome in the value dimension, and $G^t \equiv G^t(\tilde{t} \mid \sigma^s, \sigma^r, (v^s, t^s))$ her expectations regarding the outcome in the transfer dimension. For a given sender strategy σ^s, the receiver's strategy σ^r is a personal equilibrium (PE) if it is optimal for her in period 2 to always follow this plan. Moreover, strategy σ^r is a preferred personal equilibrium (PPE) if it is a PE and there is no alternative PE that yields her a higher expected payoff in period 1. An equilibrium of the game is given by a perfect Bayesian equilibrium where the receiver's strategy constitutes a preferred personal equilibrium. We state these definitions formally.

Definition 1. *For a given sender strategy σ^s, the receiver's strategy σ^r is a personal equilibrium (PE) if for any v^o and sender offer (v^s, t^s) we have*

$$U(\sigma^r(v^o, v^s, t^s) \mid G^v, G^t) \geq U(v, t \mid G^v, G^t)$$

at each available option $(v, t) \in \{(v^s, t^s), (v^o, 0)\}$. For a given sender strategy σ^s, the receiver's strategy σ^r is a preferred personal equilibrium (PPE) if it is a personal equilibrium and for any sender offer (v^s, t^s) we have

$$\mathbb{E}_{\hat{F}}[U(\sigma^r(v^o, v^s, t^s) \mid G^v, G^t)] \geq \mathbb{E}_{\hat{F}}[U(\tilde{\sigma}^r(v^o, v^s, t^s) \mid \tilde{G}^v, \tilde{G}^t)]$$

at any alternative personal equilibrium $\tilde{\sigma}^r$.

[6]In this paper, we restrict attention to pure strategies for tractability reasons. This assumption is not without loss of generality. For example, in Heidhues and Kőszegi's (2014) model of sales, the monopolist benefits from committing to a non-trivial distribution over prices.

Definition 2. *The triple* $\sigma = (\sigma^s, \sigma^r, \hat{F})$ *is a perfect Bayesian equilibrium if, for any outside option value* $v^o \in [0, 1]$, *the sender's offer* $\sigma^s(v^o)$ *maximizes his expected payoff for given* σ^r, *strategy* σ^r *is a PPE for given* σ^s, *and* $\hat{F}$ *is derived from* σ^s *and Bayes' rule whenever possible.*

This model basically describes a signaling game in which the sender (potentially) signals his private information about the receiver's outside option through the early offer to the receiver. The setting captures different strategic situations.

Shopping Example (Continuation). The outside option is produced by many firms in Bertrand competition and the sender has a competitive advantage through lower production costs. In contrast to the receiver, he always knows which products are available in the market. The regular value is product quality and the transfer a price reduction. Consider the following parametrization: The outside option has regular value $\zeta + v^o$ and production costs $c^o = \zeta$ so that the competitive price equals $p^o = \zeta$. The sender's product has regular value $\zeta + v^s$, production costs $c^s = \zeta + v^s - 1$, and price $p^s = \zeta - t^s$. If the receiver accepts his offer, the sender's payoff is $p^s - c^s = 1 - v^s - t^s$. This parametrization is equivalent to our setting for any value $\zeta \in \mathbb{R}$. We therefore can normalize $\zeta = 0$ without loss of generality.

Alternatively, the regular value is a price reduction and the transfer represents product quality. The sender can then offer a product with higher quality (and price) than the rest of the market. The corresponding parametrization would be as follows: The price and production costs of the outside option are $p^o = c^o = \zeta' - v^o$. The receiver's payoff in the transfer dimension from the outside option is u so that her consumption utility from the outside option is $u - p^o$. The price of the sender's offer is $p^s = \zeta' - v^s$ and the production costs are $c^s = \zeta' - 1 + t^s$. The receiver's payoff in the transfer dimension from the sender's offer is $u + t^s$ so that her consumption utility from the sender's offer is $u + t^s - p^s$. This parametrization is equivalent to our setting for any values $u, \zeta' \in \mathbb{R}$. In our model, we normalize $u = \zeta'$.

Negotiation Example (Continuation). Our setting also captures a bargaining situation in which the sender and the receiver have to divide a pie of size 1 among themselves. The receiver's bargaining power equals v^0 (i.e., the value that she would get if no agreement is reached). The sender makes a take-it-or-leave-it offer, which may contain a feature that the outside option (with certainty) does not have. The sender is experienced and knows which deal the receiver could get elsewhere if negotiations break down, while the receiver has to find out this information after observing the sender's proposal. Regular value and transfer capture two different outcome dimensions, e.g., the quality of a performance and customer support.

4 Signaling Equilibria

We begin the equilibrium analysis with two definitions: The sender's offer (v^s, t^s) is called inferior at outside option value v^o if $v^s + t^s < v^o$. Further, we say that the sender benefits from making early offers if the receiver accepts an inferior option at any positive outside option value $v^o > 0$. In this section, we study under what circumstances there exists an equilibrium in which the sender benefits from making early offers. In Subsection 4.1, we first discuss the benchmark case when the receiver is loss neutral and then examine some preliminary results for a loss-averse receiver. In Subsection 4.2, we state the main result and explain the structure of signaling equilibria in which the sender persuades the receiver to accept an inferior offer at any positive outside option value. Finally, in Subsection 4.3, we examine the properties of sender-preferred equilibria.

4.1 Preliminaries

We consider first the benchmark case when the receiver is loss neutral, $\lambda = 1$. In this case, she is not bothered by gain-loss sensations and accepts a sender's offer (v^s, t^s) only if $v^s + t^s \geq v^o$. In equilibrium, the sender will then, for any $v^o < 1$, make an offer with $v^s + t^s = v^o$ so that his profit equals $1 - v^o$. Making early offers has no particular value for the sender in this setting and there is no scope for persuasion through signaling.

In the following, we focus on the case when the receiver is loss averse, $\lambda > 1$. We obtain the following observation: An equilibrium in which the sender benefits from making early offers cannot be a pooling or a separating equilibrium. First, a pooling equilibrium does not exist: In a pooling equilibrium, the sender would make the same offer (v^s, t^s) at every outside option value. Thus, the only offer the sender would be willing to make in such an equilibrium is the zero offer $(v^s, t^s) = (0, 0)$. An offer $(\hat{v}^s, \hat{t}^s)$ with $\hat{v}^s > 0$ or $\hat{t}^s > 0$ could not be an equilibrium offer in a pooling equilibrium since at sufficiently low values of the outside option the sender would have an incentive to make a less generous offer. For example, at $v^o < \hat{v}^s + \hat{t}^s$ the sender would benefit from offering $(v^o + \varepsilon, 0)$ with $\varepsilon > 0$ and $v^o + \varepsilon < \hat{v}^s + \hat{t}^s$ (note that in period 2 the receiver would strictly prefer $(v^o + \varepsilon, 0)$ to her outside option, regardless of her reference point). However, the receiver would not accept the zero offer in period 2 as long as her outside option value is positive.[7]

Next, a separating equilibrium does exist. However, in any such equilibrium, there is no

[7] A pooling equilibrium in which the sender benefits from making early offers may exist if the sender were allowed to make offers (v^s, t^s) with regular value $v^s > 1$ and transfer $t^s < -1$. The former requirement is necessary since we must have $v^s > v^o$ for any possible v^o to ensure the acceptance of the sender's offer. The latter requirement is needed so that we can have $v^s + t^s \leq 0$. In the following, we do not consider this case in order to focus on signaling equilibria.

scope for exploiting the receiver's loss aversion through early offers. In a separating equilibrium, the receiver fully infers her outside option value from the sender's offer. Suppose that at some value $v^o \in (0,1)$ the sender's equilibrium offer is (v^s, t^s). The sender is willing to make this offer only if $v^s + t^s \leq v^o$ (observe that he could just offer the outside option, $v^s = v^o$ and $t^s = 0$, which the receiver would accept). In period 1, the receiver is willing to plan the acceptance of the sender's offer only if $v^s + t^s \geq v^o$. Hence, we must have $v^s + t^s = v^o$. Now, in period 2, the receiver indeed accepts the sender's offer if

$$v^s + t^s \geq v^o + \eta(v^o - v^s) - \eta\lambda t^s. \tag{5}$$

Since $\eta > 0$ and $\lambda > 1$, this inequality implies the following: If $t^s > 0$, the receiver would accept offer (v^s, t^s) in period 2 even if her outside option value is slightly larger than v^o to avoid the loss in the transfer dimension. The sender would then have an incentive to offer (v^s, t^s) at other outside option values as well. Thus, we must have $t^s = 0$ and $v^s = v^o$, so that the sender does not benefit from making early offers.[8] An equilibrium in which the sender persuades the receiver to accept an inferior offer at any $v^o > 0$ therefore must be semi-separating.

In a semi-separating equilibrium, the sender's offer can be informative about the receiver's outside option without revealing its exact value. Suppose that if the receiver gets an early offer (v^s, t^s), this informs her that her outside option is located in the non-empty, open set $V \subset [0,1]$. Define $\underline{v} = \inf(V)$ and $\bar{v} = \sup(V)$; we will use this notation throughout the paper. The receiver's PE then must be a cut-off equilibrium. The reason for this is that, at any given plan σ^r, the receiver's utility from accepting the offer (v^s, t^s) is constant, while her utility from accepting the outside option strictly increases in v^o. Hence, for any PE, there exists a value $v^* \in [\underline{v}, \bar{v}]$ so that the receiver chooses the outside option if $v^o > v^*$, and accepts the sender's offer if $v^o \leq v^*$. In general, there can be multiple PEs and it could be cumbersome to determine the PPE. However, if F is weakly convex, we obtain a result that substantially simplifies the analysis.

Lemma 1. *Let F be weakly convex. Consider any sender strategy σ^s where the sender makes the offer (v^s, t^s) if and only if $v_o \in V \subset (0,1)$, and assume that $v^s + t^s \leq \underline{v}$. Any plan σ^r that maximizes the receiver's expected payoff in period 1 at σ^s and (v^s, t^s) then specifies either (i) to always accept (v^s, t^s) when $v_o \in V$, or (ii) to always accept the outside option when $v_o \in V$.*

[8] To prove the existence of a separating equilibrium, we can use the following off-equilibrium beliefs: If the receiver observes an offer (v^s, t^s) with $t^s > 0$, she believes in period 1 that $v^o = 1$ with certainty. If $v^s + t^s < 1$, she plans to accept the outside option. It is then impossible for the sender to make an off-equilibrium offer that the receiver accepts and that offers less consumption value than the outside option.

Lemma 1 implies the following: To show that the plan[9] "accept (v^s, t^s) when $v_o \in V$" is a PPE after offer (v^s, t^s) is made, we only have to make sure that it is a PE and that it is weakly better for the receiver than the plan "accept the outside option when $v_o \in V$." In the proof of Lemma 1, we show that plans with intermediate cut-off levels $v^* \in (\underline{v}, \bar{v})$ do not maximize the receiver's expected payoff. Such plans generate gain-loss sensations in both the regular value and the transfer dimension. Since, by assumption, we have $v^s + t^s \leq \underline{v}$, they also do not generate more consumption utility than the plan "accept the outside option when $v_o \in V$." The assumption on the distribution F then ensures that either the certain rejection or the certain acceptance of the sender's offer (or both plans) maximize the receiver's expected payoff.

4.2 Main Result

When the sender makes an early offer (v^s, t^s) where the receiver knows that its total value $v^s + t^s$ is lower than that of any possible outside option, why should the receiver plan to accept it? For loss-averse receivers there is an important reason why planning to accept such an offer can be optimal. In period 1, she would then enjoy peace of mind as she will not be exposed to gain-loss sensations in period 2. Of course, accepting (v^s, t^s) must also be optimal in period 2, so the total value $v^s + t^s$ of the sender's offer cannot be too small relative to the outside option value v^o. In an equilibrium in which the sender persuades the receiver to accept an inferior offer at any $v^o > 0$, these forces must be balanced.

We show that there can exists an equilibrium where, at any positive outside option value $v^o > 0$, the sender makes (and the receiver accepts) an inferior offer (v^s, t^s). To state this result and to simplify the subsequent discussion, we refer to a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$ and define $\underline{v}_i = \inf(V_i)$ and $\bar{v}_i = \sup(V_i)$ for each interval V_i. Throughout, we assume that the sequence $\{V_i\}_{i\in\mathbb{N}}$ partitions the unit interval, and that intervals are descending, in the sense that $\underline{v}_i = \bar{v}_{i+1}$. We now can state our main result.

Proposition 1 (Signaling Equilibria)**.** *If $\eta(\lambda - 1) > 3$ and F is weakly convex, an equilibrium exists in which the sender persuades the receiver through signaling to accept an inferior offer at each outside option value $v^o > 0$. Any such equilibrium is characterized by a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$ so that the sender makes an offer (v_i^s, t_i^s) with total value $v_i^s + t_i^s < \underline{v}_i$ and positive transfer $t_i^s > 0$ if $v^o \in V_i$; the receiver always accepts this offer. If $\eta(\lambda - 1) < 3$, there exists no such equilibrium.*

The equilibrium suggested in Proposition 1 is a signaling equilibrium in which the receiver learns from an early offer about the interval in which her outside option value is located. It is

[9]This is not a fully specified strategy σ^r. Throughout the paper, we will use this "reduced" description of a strategy whenever it is not necessary to specify all details of the "complete" strategy.

shaped by three forces: the uncertainty effect, the attachment effect, and the sender's incentive to make offers that are as low as possible but are still accepted by the receiver. We explain each force in detail and elaborate what it implies for the structure of the signaling equilibrium.

The Uncertainty Effect. Suppose the receiver gets an offer (v_i^s, t_i^s) that informs her that her outside option value is in the interval V_i, with $\underline{v}_i = \inf(V_i)$ and $\bar{v}_i = \sup(V_i)$. By Lemma 1, the plan "accept (v_i^s, t_i^s) when $v_o \in V_i$" is a PPE if it is a PE and if its expected payoff in period 1 exceeds the expected payoff from the plan "accept the outside option when $v_o \in V_i$." The uncertainty effect implies that the latter requirement can be met even if accepting the outside option generates strictly more consumption utility than $v_i^s + t_i^s$. The reason is that the plan "accept the outside option when $v_o \in V_i$" has the potential for disappointments, that is, the realized outside option value may be close to the lower bound $\underline{v}_i$ in which case the receiver experiences a loss (relative to higher values of the outside option that were possible ex ante). Formally, the receiver weakly prefers the plan "accept (v_i^s, t_i^s) when $v_o \in V_i$" to "accept the outside option when $v_o \in V_i$" if[10]

$$v_i^s + t_i^s \geq \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) v \, \mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v. \tag{6}$$

Whether this inequality is satisfied or not depends on the distribution over possible outside option values $\hat{f}(v)$. Given that F is weakly convex, the distribution $\hat{F}(v)$ that minimizes the right-hand side of inequality (6) is the uniform distribution (we show this formally in the proof of Proposition 1). The intuition is that this distribution "maximizes" the uncertainty the receiver is exposed to. If $\hat{F}(v)$ is indeed a uniform distribution, then inequality (6) is satisfied if only if $\eta(\lambda - 1) > 3$ and the value $\underline{v}_i - (v_i^s + t_i^s)$ is sufficiently small.

Note that the first statement of Proposition 1 holds for all weakly convex distributions. Hence, we need a further element to ensure that the uncertainty effect unfolds for the full range of positive outside option values. Observe that, by continuity, when we make the interval $(\underline{v}_i, \bar{v}_i)$ small, then the updated density $\hat{f}(v)$ approaches a uniform distribution since $\hat{f}(\underline{v}_i) \to \hat{f}(\bar{v}_i)$; see Figure 2 for an illustration. Hence, the threshold $\eta(\lambda - 1) > 3$ holds for all weakly convex distributions F (with strictly positive and bounded density f on the unit interval) since we can always choose the intervals in $\{V_i\}_{i \in \mathbb{N}}$ small enough such that inequality (6) is satisfied for some offer (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$.

[10] An intuitive way to understand the gain-loss term on the right-hand side of inequality (6) is as follows: Take two values $\tilde{v}, v \in (\underline{v}_i, \bar{v}_i)$ with $\tilde{v} > v$. With "probability" $\hat{f}(\tilde{v})$ (resp. $\hat{f}(v)$) the outcome in the value dimension is $\tilde{v}$ (resp. v) and with the same "probability" the reference-point in the value dimension is $\tilde{v}$ (resp. v). Hence, with "probability" $\hat{f}(v)\hat{f}(\tilde{v})$ the outcome is $\tilde{v}$, while the reference point equals v, so that the receiver experiences a gain of $\eta(\tilde{v} - v)$; with the same "probability" outcome and reference point are reversed, so that the receiver experiences a loss of $\eta\lambda(\tilde{v} - v)$. The net effect is therefore $-\eta(\lambda - 1)(\tilde{v} - v)$.

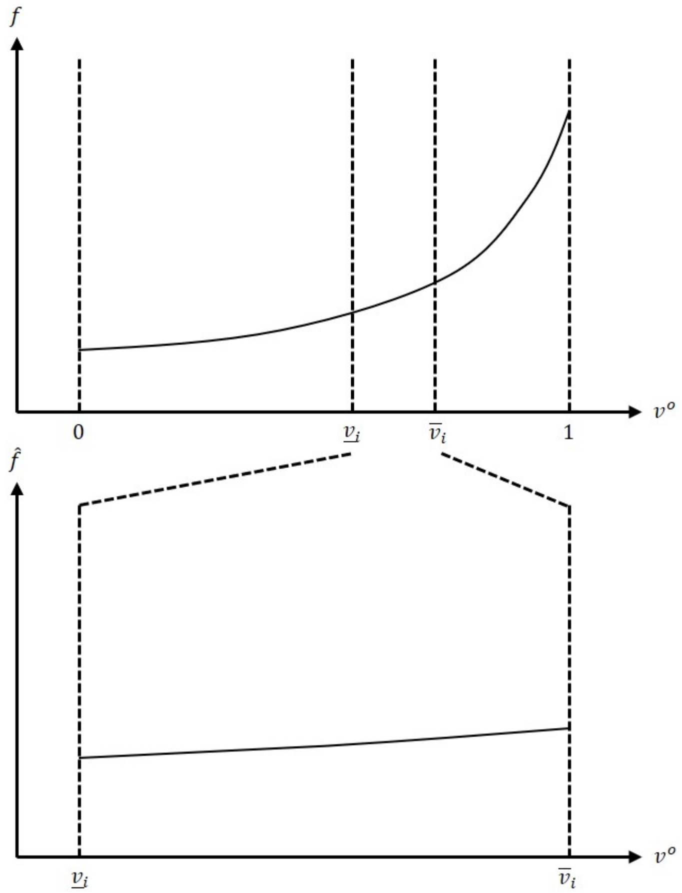

Figure 2: The updated distribution function $\hat{F}$ approaches the uniform distribution when its support $[\underline{v}_i, \bar{v}_i]$ becomes small. We exploit this effect in the proof of Proposition 1 to show that an uncertainty effect always can occur if $\eta(\lambda - 1) > 3$.

The Attachment Effect. We consider again the situation where the receiver gets an offer (v_i^s, t_i^s) that informs her that her outside option value is in the interval V_i. The receiver follows the plan "accept (v_i^s, t_i^s) when $v_o \in V_i$" only if this plan is a PE. For this, it must be optimal for the receiver to accept the sender's offer in period 2 even if the outside option value equals $\bar{v}_i$. Given the expectations induced by the plan "accept (v_i^s, t_i^s) when $v_o \in V_i$" this is the case if and only if

$$v_i^s + t_i^s \geq \bar{v}_i + \eta(\bar{v}_i - v_i^s) - \eta\lambda t_i^s. \tag{7}$$

If the inequalities in (6) and (7) are satisfied, then accepting (v_i^s, t_i^s) at all outside option values $v^o \in V_i$ is a PPE for the receiver. From inequality (7) we can make two important observations. First, the payoff-maximizing way for the sender to make an offer that satisfies inequality 7 is to create the total value only through the transfer t_i^s. Accepting the outside option implies losing the transfer, which through loss aversion is particularly painful for the receiver; we can observe this from the term $\eta\lambda t_i^s$. As a result, the receiver is "attached" to the offer. Second, and relatedly, inequality (7) puts an upper bound on the length of the interval V_i. If the total value is smaller than the lowest possible outside option value, $v_i^s + t_i^s < \underline{v}_i$, then both inequalities taken together imply that we must have $\underline{v}_i \geq \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$. As we will see in the next subsection, this restriction may define the shape of the equilibrium that is optimal for the sender.

Sender Incentives. The proposed equilibrium is a signaling equilibrium only if the sender has an incentive to make the offer (v_i^s, t_i^s) when $v^o \in V_i$. Specifically, he must not have an incentive to make this offer when $v^o > \bar{v}_i$. Offers and intervals therefore must be chosen such that the receiver rejects an offer (v_i^s, t_i^s) if her true outside option (unexpectedly) exceeds $\bar{v}_i$. Note that upon receiving offer (v_i^s, t_i^s) the receiver expects that $v^o \in V_i$ and that she accepts the offer in period 2. It is then optimal for her to reject (v_i^s, t_i^s) for any $v^o > \bar{v}_i$ if and only if

$$v_i^s + t_i^s \leq \bar{v}_i + \eta(\bar{v}_i - v_i^s) - \eta\lambda t_i^s. \tag{8}$$

The right-hand side of this inequality is the expected payoff from accepting the outside option when $v^o = \bar{v}_i$. Since the inequality in (7) also needs to be satisfied to ensure sufficient attachment, the offer (v_i^s, t_i^s) for an interval V_i must be chosen such that

$$v_i^s + t_i^s = \bar{v}_i + \eta(\bar{v}_i - v_i^s) - \eta\lambda t_i^s. \tag{9}$$

For given v_i^s and interval V_i the transfer t_i^s must therefore satisfy $t_i^s = \frac{1+\eta}{1+\eta\lambda}\bar{v}_i - \frac{1+\eta}{1+\eta\lambda}v_i^s$. Thus, the sender can reduce the total value $v_i^s + t_i^s$ while respecting (9) by substituting transfer t_i^s for regular value v_i^s. The scope for this substitution may be constrained by the fact that it still must be optimal for the receiver to plan acceptance in period 1. Formally, this means that the inequality in (6) must be satisfied as well. Observe that the smallest possible total value that the sender can offer to the receiver according to the condition in (9) is $v_i^s + t_i^s = \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$. However, if $\underline{v}_i$ is relatively close to $\bar{v}_i$ – which may be necessary if $\eta(\lambda-1)$ is sufficiently close to 3 and F is not the uniform distribution (see the discussion of the uncertainty effect above) – then according to the condition in (6) the total value $v_i^s + t_i^s$ must be relatively close to $\bar{v}_i$. Since $\bar{v}_i > \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$, it may be necessary to make offers with both positive regular value v_i^s and positive transfer t_i^s to satisfy both the condition in (6) and that in (9): The transfer t_i^s is then positive to exploit the attachment effect, and the value v_i^s is positive to enable credible signaling.

The uncertainty effect, the attachment effect, and the sender's incentives constrain the intervals $\{V_i\}_{i\in\mathbb{N}}$ and the offers $\{(v_i^s, t_i^s)\}_{i\in\mathbb{N}}$ of an equilibrium in which the sender persuades the receiver to accept an inferior offer at all positive outside option values. To construct such an equilibrium, it then only remains to fix off-equilibrium beliefs. It must be optimal for the receiver to reject any off-equilibrium offer $(\tilde{v}_i^s, \tilde{t}_i^s)$ when $\tilde{v}_i^s + \tilde{t}_i^s < v^o$. One option is to assume "optimistic beliefs", that is, the receiver believes in period 1 that her outside option value is maximal, $v^o = 1$, after receiving an off-equilibrium offer $(\tilde{v}_i^s, \tilde{t}_i^s)$.[11] Given this belief, it is then indeed optimal to reject this offer if $\tilde{v}_i^s + \tilde{t}_i^s < v^o$.

The last part of Proposition 1 shows that the sender cannot benefit from early offers if F is weakly convex and $\eta(\lambda - 1) < 3$. The reason for this is that the uncertainty effect is then no longer strong enough so that the receiver would accept any offer (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$, irrespective of how we choose V_i. The inequality in (6) would not be satisfied for any such offer. We show in Section 5 that this changes once we have uncertainty in multiple dimensions.

4.3 Sender-Preferred Equilibria

Like in most signaling games, there are many equilibria in our setting. Classic refinements like the Intuitive Criterion (Cho and Kreps 1987) or Undefeated Equilibrium (Mailath et al. 1993) do not reduce the number of equilibria in a meaningful way in our case. The reason is that there can be continuum of offers where each offer is optimal at a certain outside option value given that the receiver plans their acceptance in period 1. In order to select between equilibria, we examine equilibria in which the sender earns the highest possible ex ante expected payoff, that is, the "sender-preferred equilibrium."[12]

For general distributions F, the sender-preferred equilibrium is the solution to a complex maximization problem where one has to find the optimal length of the intervals $\{V_i\}_{i \in \mathbb{N}}$. As discussed above, the length of an interval must be chosen so that the uncertainty effect is sufficiently strong. Hence, the optimal configuration of intervals in general depends on the local properties of the distribution F. Nevertheless, we can show that the sender-preferred equilibrium must have an interval structure and positive outcomes in the transfer dimension as indicated in Proposition 1.

We can say more about the sender-preferred equilibrium when F is the uniform distribution on the unit interval. This assumption simplifies the problem substantially. If F is the uniform distribution, the updated distribution $\hat{F}$ is also (piecewise) uniform on its support at any equilibrium offer. Given that $\eta(\lambda - 1) > 3$, there is then enough uncertainty in each interval V_i so that the receiver would accept an offer that is worse than the lowest value in the interval. This allows us to characterize the sender-preferred equilibria.

To this end, we define an upper bound on $\underline{v}_i$ for a given value $\bar{v}_i$. Suppose that offer (v_i^s, t_i^s) informs the receiver that her outside option value is in the interval V_i. Recall that the smallest total value the sender needs to offer so that the receiver always accepts (v_i^s, t_i^s) in period 2 is $\frac{1+\eta}{1+\eta\lambda}\bar{v}_i$. The corresponding offer creates utility only through the transfer, i.e.,

[11] This is the analog to "pessimistic beliefs" which are frequently assumed in job-market signaling models in order to motivate the equilibrium strategies.

[12] Alonso and Câmara (2018) use a similar solution concept.

$(v_i^s, t_i^s) = (0, \frac{1+\eta}{1+\eta\lambda}\bar{v}_i)$. Provided that the sender makes this offer only if $v^o \in V_i$, it is optimal for the receiver in period 1 to plan accepting $(0, \frac{1+\eta}{1+\eta\lambda}\bar{v}_i)$ if and only if this is weakly better than always accepting the outside option. This is the case if and only if

$$\frac{1+\eta}{1+\eta\lambda}\bar{v}_i \geq \frac{1}{2}(\bar{v}_i + \underline{v}_i) - \frac{1}{6}\eta(\lambda-1)(\bar{v}_i - \underline{v}_i), \tag{10}$$

where the right-hand side of this inequality equals the right-hand side of the inequality in (6) when F is the uniform distribution. It defines an upper bound on $\underline{v}_i$ for a given value $\bar{v}_i$. This upper bound equals $\Gamma(\eta,\lambda)\bar{v}_i$, where

$$\Gamma(\eta,\lambda) = \frac{6\frac{1+\eta}{1+\eta\lambda} - 3 + \eta(\lambda-1)}{3+\eta(\lambda-1)}. \tag{11}$$

The intuition is that $\underline{v}_i$ cannot be closer to $\bar{v}_i$ than $\Gamma(\eta,\lambda)\bar{v}_i$, otherwise there is too little uncertainty for the uncertainty effect to unfold. It turns out that this bound also defines the intervals in a sender-preferred equilibrium. We thus obtain the following result.

Proposition 2 (Sender-Preferred Equilibrium)**.** *If $\eta(\lambda-1) > 3$, the following statements hold.*

(i) Suppose F is weakly convex. Any sender-preferred equilibrium is then characterized by a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$ so that the sender makes an offer (v_i^s, t_i^s) with total value $v_i^s + t_i^s \leq \underline{v}_i$ and positive transfer $t_i^s > 0$ if $v^o \in V_i$; the receiver always accepts this offer.

(ii) Suppose F is the uniform distribution on the unit interval. Any sender-preferred equilibrium is then characterized by a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$, so that the sender offers $(v_i^s, t_i^s) = (0, \frac{1+\eta}{1+\eta\lambda}\bar{v}_i)$ if $v^o \in V_i$; the receiver always accepts this offer. The length of the intervals is minimal subject to the constraint that it is optimal for the receiver to plan acceptance, i.e., $\underline{v}_i = \Gamma(\eta,\lambda)\,\bar{v}_i$ for all $i \in \mathbb{N}$.

For the case of a uniform distribution the result shows that, in the sender-preferred equilibrium, the sender makes for each interval V_i the least generous offer that is still accepted, and reduces the offered total value as quickly as possible as the outside option value decreases. Figure 3 displays a sender-preferred equilibrium for the case $\eta = 2$ and $\lambda = 3$.

The proof of Proposition 2 is not as straightforward as it might seem. There is a trade-off between the total value $v_i^s + t_i^s$ that is offered in an interval V_i and the length of this interval. The higher the total value is, the smaller the interval can be (so that a further reduction of the total value is possible in the next interval). It turns out, however, that with a uniform distribution the sender's expected expenses are minimal if at any given interval he makes the least generous offer and, given this fact, intervals are as short as possible.

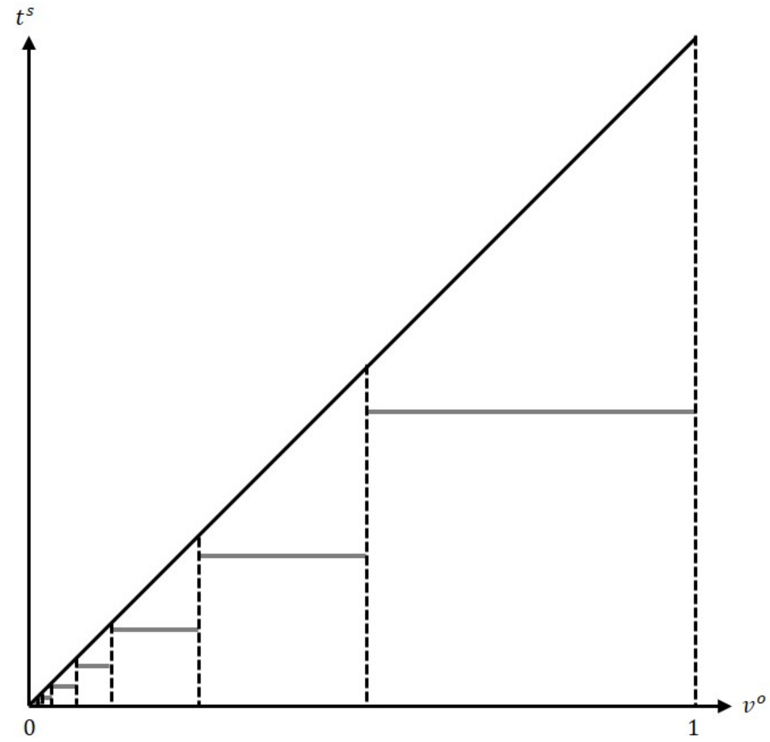

Figure 3: The sender-preferred equilibrium for $\eta = 2$ and $\lambda = 3$ when F is the uniform distribution. The 45-degree line indicates offers with total value v^o. The dotted vertical lines indicate the bounds of the intervals $\{V_i\}_{i\in\mathbb{N}}$. The gray lines indicate, for each interval V_i, the offered transfer t_i^s (and hence also the offered total value).

The factor $\Gamma(\eta, \lambda)$ defines how informative the sender-preferred signaling equilibrium is. The closer $\Gamma(\eta, \lambda)$ is to 1, the shorter the intervals, and the more informative is the equilibrium. $\Gamma(\eta, \lambda)$ is not monotone in the degree of loss aversion λ. The reason is that there are two competing forces that influence $\Gamma(\eta, \lambda)$. On the one hand, the larger λ is, the smaller is the offered transfer $t_i^s = \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$ in interval V_i. This in turn requires more uncertainty. On the other hand, the larger λ is, the less uncertainty is needed for the uncertainty effect to unfold at a given offer. Observe that $\Gamma(\eta, \lambda)$ strictly increases in λ if λ is sufficiently large, and approaches unity when the receiver becomes very loss averse.

5 Uncertainty in Multiple Dimensions

In our baseline model, the sender only benefits from making early offers if the receiver's loss aversion parameters η, λ are large enough so that $\eta(\lambda - 1) > 3$. The reason for this requirement is that there is uncertainty only in one outcome dimension (i.e., the regular value dimension). This has the following consequence: If for a given upper bound of the potential outside option values $\bar{v}$ the sender wants to make a more generous offer – that is, move the total value of the offer $v^s + t^s$ closer to $\bar{v}$ – this reduces in a signaling equilibrium the uncertainty about the outside option value and hence the strength of the uncertainty effect. As $v^s + t^s$ approaches $\bar{v}$, the uncertainty effect vanishes. Therefore, if there is uncertainty only in one outcome dimension, then the attractiveness of an offer relative to the outside option and the extent of the uncertainty effect are tightly linked.

Degrees of loss aversion that satisfy $\eta(\lambda - 1) > 3$ are empirically relevant (e.g., von Gaudecker et al. 2011) and the uncertainty effect has been found in numerous settings (as discussed in Section 2). However, in theoretical applications of expectation-based loss-averse preferences, the assumed levels of loss aversion are typically smaller. In this section, we present an extension of the model in which our main results – persuasion through signaling as well as interaction of attachment and uncertainty effect – obtain for all loss aversion parameters η, λ that satisfy $\eta(\lambda - 1) > 0$. The idea behind this extension is that there is uncertainty about the outside option in multiple dimensions that are payoff relevant for the receiver. For example, if the outside option is a product, there can be uncertainty about product specifications, delivery times, customer support, warranties, and so forth. The receiver may not evaluate the joint value of these different attributes, but narrowly brackets them so that gain-loss sensations occur in multiple dimensions. As a result, some uncertainty about the specification of the outside option remains even if the receiver knows the exact value of the outside option v^o. In a signaling equilibrium, this relaxes the link between the attractiveness of an offer and the strength of the uncertainty effect.

In the following, we extend our baseline model by assuming that the receiver faces uncertainty in multiple dimensions. The expectations-based loss-aversion framework of Kőszegi and Rabin (2006) explicitly allows for multiple value dimensions. We implement them in two ways that allow us to keep the model tractable. First, in Subsection 5.1, we consider a setting where uncertainty occurs in extra-dimensions so that the overall consumption utility from the sender's offer and the outside option remain unchanged. Second, in Subsection 5.2, we consider a setting with two value dimensions that both matter for overall consumption utility.

5.1 Uncertainty in Extra-Dimensions

Updated Setting. We consider the same model as in Section 3, with the difference that any option has values in two extra-dimensions, the x-dimension and the y-dimension. The sender's offer is now given by (v^s, t^s, x^s, y^s), where $x^s = y^s = 0$. We can interpret these values as non-strategic design choices. As before, the sender chooses the regular value v^s and the transfer t^s of his early offer. The outside option is characterized by the vector $(v^o, 0, x^o, y^o)$, where v^o is again the outside option value and distributed according to F on the unit interval. With probability $\frac{1}{2}$ we have $x^o = \xi$ and $y^o = -\xi$ for some value $\xi > 0$ (state 1), and with probability $\frac{1}{2}$ we have $x^o = -\xi$ and $y^o = \xi$ (state 2). The parameter ξ captures the level of uncertainty in the extra-dimensions the receiver is exposed to if she plans to accept the outside option. For

$\xi = 0$ the new version of the model is equivalent to the baseline model. The consumption utility from any option is $v + t + x + y$. Hence, as in the baseline model, the consumption utility from the sender's offer again equals $v^s + t^s$ and the consumption utility from the outside option equals v^o. If in period 1 the consumer expects to accept an offer $(\tilde{v}, \tilde{t}, \tilde{x}, \tilde{y})$ with certainty, and ends up accepting option (v, t, x, y), her utility is

$$U(v, t, x, y \mid \tilde{v}, \tilde{t}, \tilde{x}, \tilde{y}) = v + t + x + y + \mu(v - \tilde{v}) + \mu(t - \tilde{t}) + \mu(x - \tilde{x}) + \mu(y - \tilde{y}). \qquad (12)$$

Therefore, the consumer may experience gain-loss sensations in four dimensions (instead of two). The rest of the model proceeds as before. We assume that ξ is not too large so that $\eta(\lambda - 1)\xi < 1 + \eta$. For convenience, we denote in the following a sender's offer by (v^s, t^s) instead of (v^s, t^s, x^s, y^s).

Signaling Equilibria. We first adapt Lemma 1 to the new environment. Again, the receiver's PE must be a cut-off plan. This plan can be contingent on the state, i.e., the receiver may adopt different cut-off levels in the two states. However, in the proof of the next result, we show that the receiver cannot increase her expected payoff by adopting a plan with state-contingent cut-off levels. Hence, as in Lemma 1, a cut-off plan that maximizes her expected payoff in period 1 is either "always accept the sender's offer when $v_o \in V$" or "always accept the outside option when $v_o \in V$." This result is independent of the level of uncertainty ξ in the extra-dimensions.

Lemma 2. *Consider the model with uncertainty in extra-dimensions. Let F be weakly convex. Consider any sender strategy σ^s where the sender makes the offer (v^s, t^s) if and only if $v_o \in V \subset (0, 1)$, and assume that $v^s + t^s \leq \underline{v}$. Any plan σ^r that maximizes the receiver's expected payoff in period 1 at σ^s and (v^s, t^s) then specifies either (i) to always accept (v^s, t^s) when $v_o \in V$, or (ii) to always accept the outside option when $v_o \in V$.*

Next, we characterize under what circumstances an equilibrium exists in which the sender persuades the receiver to accept an inferior offer at any positive outside option value. Due to the uncertainty in the extra-dimensions, a plan that involves accepting the outside option now implies even more gain-loss sensations. This increases the scope for the uncertainty effect. It is therefore conceivable that the critical threshold for the parameter $\eta(\lambda - 1)$ decreases as the uncertainty parameter ξ increases. However, we obtain an even stronger result. As long as $\eta(\lambda - 1) > 0$ and ξ is positive, there exists an equilibrium in which the sender benefits from making early offers.

Proposition 3 (Signaling Equilibria with Multi-Dimensional Outside Option Values)**.** *Consider the model with uncertainty in extra-dimensions. If $\eta(\lambda - 1) > 0$, $\xi > 0$ and F is weakly convex, an equilibrium exists in which the sender persuades the receiver through signaling to accept an inferior offer at each outside option value $v^o > 0$.*

In the proof of Proposition 3, we again construct the desired equilibrium through a sequence of disjoint intervals $\{V_i\}_{i=1,\dots,n}$ for some finite n, so that the sender makes an offer (v_i^s, t_i^s) with total value $v_i^s + t_i^s < \underline{v}_i$ if $v^o \in V_i$. There are two new elements here. First, as we lower the length of an interval $\bar{v}_i - \underline{v}_i$, the magnitude of expected gain-loss sensations converges against a positive value, and not against zero as in the baseline model. This effect is due to the uncertainty in the extra-dimensions. Hence, for any given loss aversion parameters η, λ that satisfy $\eta(\lambda - 1) > 0$, if the interval V_i is short enough, we can find an offer (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$ so that the receiver's expected payoff in period 1 is maximal if she plans to accept this offer as long as $v^o \in V_i$. The second new element is that we no longer need an infinite sequence of intervals. If the outside option value is sufficiently small, then it is possible that in equilibrium the receiver is willing to accept the least generous offer $(v^s, t^s) = (0, 0)$. The reason is that (unexpectedly) accepting the outside option always creates a loss sensation in one extra-dimension.

A crucial difference between Proposition 1 and Proposition 3 is that, with the two extra-dimensions, an equilibrium in which the sender persuades the receiver to accept an inferior option does not necessarily have an interval-structure or positive outcomes in the transfer dimension. This, however, changes when we consider sender-preferred equilibria. We can show that if ξ is small enough, then a sender-preferred equilibrium exhibits both bunching of outside option values and positive outcomes in the transfer dimension, at least for the highest outside option values. The next result states this observation formally.

Proposition 4 (Sender-Preferred Equilibrium, Multi-Dimensional Outside Option Values)**.** *Consider the model with uncertainty in extra-dimensions. Suppose that $\eta(\lambda - 1) > 0$, $\xi > 0$, and F is weakly convex. If for given values η, λ the parameter ξ is small enough, then in a sender-preferred equilibrium the sender offers the same total value with a positive outcome in the transfer dimension for all $v^o \in (\underline{v}, 1]$ for some $\underline{v} < 1$.*

The intuition for this result will become clear in the discussion of the next proposition. In general, the exact shape of sender-preferred equilibria depends on the loss aversion parameters η, λ, the uncertainty parameter ξ, and the distribution over outside option values F. Nevertheless, for the special case of a uniform distribution F, we can characterize sender-preferred equilibria if the loss aversion parameters η, λ satisfy

$$\frac{1+\eta}{1+\eta\lambda} < \frac{1}{2} + \frac{1}{6}\eta(\lambda - 1). \tag{13}$$

This inequality holds if λ is large enough for given η (e.g., $\lambda > 2$ at $\eta = 1$). If additionally ξ is small enough, a sender-preferred equilibrium has the following structure.

Proposition 5 (Sender-Preferred Equilibrium, Multi-Dimensional Outside Option Values, Uniform Distribution)**.** *Consider the model with uncertainty in extra-dimensions. Suppose that $\eta(\lambda - 1) > 0$, $\xi > 0$, $\eta\lambda\xi < 1$, the condition in (13) is satisfied, and that F is the uniform distribution on the unit interval. Any sender-preferred equilibrium is then characterized by a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$ that partitions the interval $[\eta\lambda\xi, 1]$. In this equilibrium, the sender makes the following offers:*

(i) If $v^o \leq \frac{\eta(\lambda-1)}{1+\eta}\xi$, the sender offers $(0,0)$.

(ii) If $\frac{\eta(\lambda-1)}{1+\eta}\xi < v^o \leq \eta\lambda\xi$, the sender offers $(0, t^s)$ with

$$t^s = \frac{(1+\eta)v^o - \eta(\lambda-1)\xi}{1+\eta\lambda}.$$

(iii) If $v^o \in V_i \subset [\eta\lambda\xi, 1]$, the sender offers $(0, t_i^s)$ with

$$t_i^s = \frac{(1+\eta)\bar{v}_i - \eta(\lambda-1)\xi}{1+\eta\lambda}.$$

The receiver always accepts the sender's offer. The length of the intervals $\{V_i\}_{i\in\mathbb{N}}$ is minimal subject to the constraint that it is optimal for the receiver to plan acceptance.

Figure 4 displays the sender-preferred equilibrium for $\eta = 1$, $\lambda = 2.25$, and $\xi = 0.2$. In this equilibrium, outside option values are grouped in three domains: small, intermediate, and large values. In the following, we explain the intuition behind the offers in each of these domains. This intuition captures the subtle interaction of uncertainty and attachment effect. At small outside option values, the sender makes the least generous offer $(0, 0)$. The receiver accepts this offer in period 2 since rejecting it would generate gain-loss sensations in the two extra-dimensions. At intermediate values v^o, the sender makes offers with positive transfers that are fully informative about the receiver's outside option value. In this domain, the uncertainty effect is created only through the extra-dimensions, while the attachment effect is generated through the transfer as well as the extra-dimensions. Both effects are relatively small (since

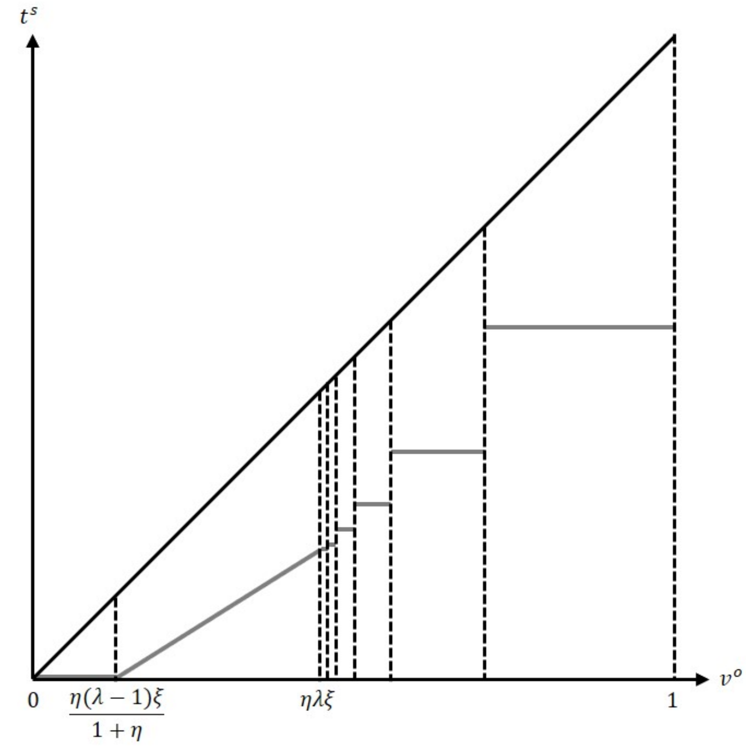

Figure 4: The sender-preferred equilibrium for $\eta = 1$ and $\lambda = 2.25$ when F is the uniform distribution and there is uncertainty $\xi = 0.2$ in two extra-dimensions. The 45-degree line indicates offers with total value v^o. The dotted vertical lines indicate the bounds of the intervals $\{V_i\}_{i\in\mathbb{N}}$ and the three cases of Proposition 5. The gray lines indicate, for each interval V_i, the offered transfer t_i^s (and hence also the offered total value).

the transfer is still relatively small) so that it is not necessary to generate additional uncertainty through the bunching of outside option values.

Finally, at large outside option values, bunching occurs. Suppose that this were not the case and that the sender would still make offers as in the intermediate domain so that they are fully informative about the outside option. The transfer then would have to be relatively large (to compensate the receiver for the outside option value). Consequently, the attachment effect – which is generated through the transfer and the extra-dimensions – would be relatively large compared to the uncertainty effect that is still created only through the extra-dimensions. Therefore, in order to reduce the total value necessary to get the receiver's acceptance, the optimal offers employ bunching to create additional uncertainty.

5.2 Uncertainty in Multiple Value Dimensions

Updated Setting. We consider a variation of the model from the previous subsection where the receiver faces uncertainty in two value dimensions. This is probably the more natural way to have uncertainty in multiple dimensions. However, it is also more challenging to describe the PPE in this setting. Hence, we only consider it in addition to the setting from the previous subsection.

The outside option value v^o now consists of two parts, $v^{o,1}$ and $v^{o,2}$, so that $v^o = v^{o,1} + v^{o,2}$. As before, v^o is distributed according to F on the unit interval. With probability $\frac{1}{2}$ we have $v^{o,1} = \frac{1}{2}v^o + \xi$ and $v^{o,2} = \frac{1}{2}v^o - \xi$ for some $\xi > 0$ (state 1), and with probability $\frac{1}{2}$ we have $v^{o,1} = \frac{1}{2}v^o - \xi$ and $v^{o,2} = \frac{1}{2}v^o + \xi$ (state 2). Thus, ξ captures the degree of uncertainty in the two value dimensions. Overall, the outside option is given by the vector $(v^{o,1}, v^{o,2}, 0)$, while the sender's offer is given by $(v^{s,1}, v^{s,2}, t^s)$. The sender offers the same outcome in the two value dimensions: If he chooses value v^s for his offer, we have $v^{s,1} = v^{s,2} = \frac{1}{2}v^s$.

If in period 1 the receiver expects to accept option $(\tilde{v}^1, \tilde{v}^2, \tilde{t})$ with certainty, and ends up accepting option (v^1, v^2, t), her utility is

$$U(v^1, v^2, t \mid \tilde{v}^1, \tilde{v}^2, \tilde{t}) = v^1 + v^2 + t + \mu(v^1 - \tilde{v}^1) + \mu(v^2 - \tilde{v}^2) + \mu(t - \tilde{t}). \tag{14}$$

The rest of the model proceeds as before. We assume that the uncertainty level ξ is small enough such that $\eta(\lambda - 1)\xi < 1 + \frac{1}{2}\eta + \frac{1}{2}\eta\lambda$. For convenience, we again denote a sender's offer by (v^s, t^s) instead of $(v^{s,1}, v^{s,2}, t^s)$.

Signaling Equilibria. We show that in this setting we again obtain a signaling equilibrium in which the sender convinces the receiver to accept an inferior option at any positive outside option value v^o. In particular, we now obtain this result without any further assumption on the distribution function F.

Proposition 6 (Signaling Equilibria with Multiple Value Dimensions)**.** *Consider the model with uncertainty in multiple value dimensions. If $\eta(\lambda - 1) > 0$ and $\xi > 0$, an equilibrium exists in which the sender persuades the receiver through signaling to accept an inferior offer at each outside option value $v^o > 0$.*

To prove this result, we employ a signaling strategy without interval-structure and without positive outcomes in the transfer dimension. Specifically, we use the following strategy. At a given outside option value v^o the sender offers $(v^s, t^s) = (v^s, 0)$ and

$$\begin{aligned} v^s &= v^o - \frac{\eta(\lambda - 1)}{1 + \frac{1}{2}\eta + \frac{1}{2}\eta\lambda}\xi \quad \text{if} \quad v^o > \frac{\eta(\lambda - 1)}{1 + \frac{1}{2}\eta + \frac{1}{2}\eta\lambda}\xi, \\ v^s &= 0 \qquad\qquad\qquad\qquad\quad \text{if} \quad v^o \le \frac{\eta(\lambda - 1)}{1 + \frac{1}{2}\eta + \frac{1}{2}\eta\lambda}\xi. \end{aligned}$$

Note that, if v^o is large enough, the receiver can infer the exact value of the outside option from the sender's offer. Nevertheless, she then still faces uncertainty about the realization in the two value dimensions. If she plans to accept the offer and – unexpectedly – accepts the outside option in period 2, her payoff is given by

$$v^o + \eta\left(\frac{1}{2}v^o + \xi - \frac{1}{2}v^s\right) - \eta\lambda\left(\frac{1}{2}v^s - \frac{1}{2}v^o + \xi\right). \tag{15}$$

The sender strategy implies that this term is equal to the value v^s. Hence, in period 2, the receiver is indifferent between accepting and rejecting the sender's offer. As we show in the proof of Proposition 6, it is indeed optimal for the receiver to plan the acceptance of the sender's offer in period 1 as long as $\eta(\lambda - 1) > 0$. Thus, the sender can benefit from making early offers to the receiver for any (positive) degree of loss aversion and any degree of uncertainty ξ. Again, both the uncertainty and the attachment effect must be strong enough to generate this outcome.

The sender-preferred equilibrium in this setting may again exhibit an interval-structure to boost the receiver's uncertainty about her outside option in period 1 (especially when ξ is small), as well as positive outcomes in the transfer dimension to strengthen the attachment effect. Unfortunately, with uncertainty in two value dimensions, we no longer obtain a simple characterization under what circumstances the receiver strategy constitutes a PPE (as we did for the previous settings in Lemma 1 and Lemma 2).

6 Extensions

We consider a number of extensions of our baseline model. In Subsection 6.1, we study how the sender-preferred equilibrium changes when the sender can commit to time-limited offers. In Subsection 6.2, we consider the case when the sender does not know the receiver's outside option with some positive probability. In Subsection 6.3, we analyze to what extent our main result holds if exchanging regular value for transfers is costly for the sender. In Subsection 6.4, we discuss the case when the sender has imperfect information about the receiver's degree of loss aversion. Finally, in Subsection 6.5, we briefly examine the welfare consequences of persuasion through early offers in our setting.

6.1 Requesting Immediate Acceptance

The result in Proposition 1 shows that the sender can persuade the receiver to accept an inferior offer even if the receiver has full information at the decision stage. This begs the question whether the sender can gain even more if the receiver has incomplete information at the point in time when she has to decide between options. For example, he could force the receiver to make a decision already in period 1. In many circumstances, sellers are doing exactly this. A classic sales tactic is to "create time pressure", i.e., urge the consumer to make a quick decision as the offered product may soon be out of stock. In the following, we briefly examine how the sender-preferred equilibrium changes when the sender has the commitment power to request immediate acceptance.[13]

We consider the same model as in Section 3 with the following change. After observing the receiver's outside option value v^o, the sender not only chooses an offer (v^s, t^s), but also decides whether or not he requests immediate acceptance. If he requests immediate acceptance, the offer is only valid in period 1. In this case, if the receiver accepts the offer in period 1, payoffs are realized and the game is over. If the receiver rejects the offer in period 1, it is no longer available in period 2 so that she has to accept her outside option. When the sender does not request immediate acceptance, the receiver can accept the offer (v^s, t^s) only in period 2 after learning her outside option value. The receiver observes whether immediate acceptance is requested or not and updates her beliefs about the outside option value accordingly.

Assume first that the sender requests immediate acceptance at any outside option value. Provided that the receiver always accepts the sender's offer, this is possible in equilibrium only if the sender always makes the least generous offer $(v^s, t^s) = (0, 0)$, regardless of the outside option value. In period 2, the receiver would not accept this offer as long as the outside option value is positive. However, in period 1, if the receiver has to decide immediately, it is optimal for her to accept offer $(0, 0)$ if

$$0 \geq \int_0^1 f(v)v \, \mathrm{d}v - \eta(\lambda - 1) \int_0^1 f(v) \int_v^1 f(\tilde{v}) (\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v. \tag{16}$$

If this inequality is satisfied, the receiver prefers to earn zero with certainty than to being exposed to the gain-loss sensations from accepting the outside option. It is satisfied if $\eta(\lambda - 1)$ is large enough. For a given distribution F over outside option values, denote by Λ_F the value of $\eta(\lambda - 1)$ so that (16) is satisfied with equality. Clearly, this value depends on F. If F is the uniform distribution, we have $\Lambda_F = 3$. If $\eta(\lambda - 1) \geq 3$, the sender then offers $(0, 0)$ and requests immediate acceptance in the sender-preferred equilibrium. From the second part of Proposition 2 we can derive by how much the sender's commitment power increases his profit: Relative to the sender-preferred equilibrium without commitment, his payoff is raised by $\frac{1+\eta}{1+\eta\lambda}\bar{v}_i$ when $v^o \in V_i$, where V_i is the interval defined in Proposition 2. Next, suppose that F is some strictly convex distribution. We then have $\Lambda_F > 3$ and we obtain a more nuanced description of the sender-preferred equilibrium. The following result is a corollary to Proposition 2.

[13]Rosato (2016) also studies the sales tactic of creating scarcity for products that are on sale ("Black Friday deals") in order to induce consumers to purchase options that offer less consumer surplus. The difference to his setting here is that the sender forces the receiver to make a decision when she does not yet know the features of all available options.

Corollary 1 (Immediate Acceptance)**.** *Consider the model in which the sender can request immediate acceptance of an offer in period 1. Let F be weakly convex.*

(i) If $3 < \eta(\lambda - 1) < \Lambda_F$, any sender-preferred equilibrium is characterized by a sequence of disjoint intervals $\{V_i\}_{i \in \mathbb{N}}$ so that the sender makes an offer (v_i^s, t_i^s) with total value $v_i^s + t_i^s \leq \underline{v}_i$ if $v^o \in V_i$; the receiver always accepts this offer. The sender may request immediate acceptance in the highest interval V_1, but not in any other interval.

(ii) If $\eta(\lambda - 1) \geq \Lambda_F$, then, in any sender-preferred equilibrium, the sender offers $(v^s, t^s) = (0, 0)$ at each outside option value and requests immediate acceptance.

We already know from Proposition 1 that if $\eta(\lambda - 1) < 3$, the sender cannot persuade the receiver to accept an inferior offer. If the value $\eta(\lambda - 1)$ is in the range between 3 and Λ_F, the sender can benefit from making early offers, but it is not optimal for him to offer $(0, 0)$ and to request immediate acceptance since the receiver would reject this offer. Instead, the sender-preferred equilibrium again has an interval structure where the equilibrium offer becomes less generous in lower intervals. The sender may request immediate acceptance only in the highest interval V_1 where he makes the most generous equilibrium offer. Otherwise, to ensure the credibility of the signal, the receiver first has to learn her true outside option value and then accepts the sender's offer. Finally, if $\eta(\lambda - 1) \geq \Lambda_F$, it is optimal for the receiver to accept a certain payoff of zero instead of accepting an outside option of uncertain value. In a sender-preferred equilibrium, the sender then realizes the maximal possible payoff.

6.2 Uncertain Outside Option Value

In our baseline model, the sender always knows the sender's outside option value. One may ask to what extent the sender can benefit from making early offers if this assumption is not satisfied. Using the analysis from Section 4, we can address this question in the following simple extension. Suppose that with probability $\beta \in (0, 1)$ the sender knows the receiver's outside option value v^o in period 1, and with probability $1 - \beta$ he only knows the prior distribution over outside option values F. All other aspects of the model remain unchanged.

We examine under what circumstances an equilibrium exists in which the sender benefits from making early offers. For the case when the sender does not observe the receiver's outside option, we say that he benefits from making early offers if the receiver accepts with certainty an offer $(\hat{v}^s, \hat{t}^s)$ that has less total value than the outside option in expectation,

$$\hat{v}^s + \hat{t}^s < \int_0^1 \hat{f}(v) v \, \mathrm{d}v. \tag{17}$$

Assume that from observing offer $(\hat{v}^s, \hat{t}^s)$ the receiver infers that the sender does not know the outside option value. In period 1, she then weakly prefers the plan "accept $(\hat{v}^s, \hat{t}^s)$ when $v^o \in [0, 1]$" to the plan "accept the outside option when $v^o \in [0, 1]$" if

$$\hat{v}^s + \hat{t}^s \geq \int_0^1 f(v)v\,\mathrm{d}v - \eta(\lambda - 1)\int_0^1 f(v)\int_v^1 f(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v}\,\mathrm{d}v. \tag{18}$$

Note that there is a range of total values $\hat{v}^s + \hat{t}^s$ that satisfy both inequality (17) and (18). Next, in period 2, the receiver is indeed willing to accept $(\hat{v}^s, \hat{t}^s)$ even at the highest possible outside option value if and only if

$$\hat{v}^s + \hat{t}^s \geq 1 + \eta(1 - \hat{v}^s) - \eta\lambda\hat{t}^s. \tag{19}$$

If the loss aversion parameters η, λ are small enough, the inequalities (17) and (19) would contradict each other. However, there is an open set of offers $(\hat{v}^s, \hat{t}^s)$ that satisfy all three inequalities (17) to (19) if F is weakly convex and $\eta(\lambda - 1) > 1 + \eta$.

We can now establish the desired result. Assume that F is weakly convex and $\eta(\lambda - 1) > \max\{1 + \eta, 3\}$. Suppose the sender offers $(\hat{v}^s, \hat{t}^s)$ if he does not know v^o and (v_i^s, t_i^s) if he knows v^o and $v^o \in V_i$. The arguments above and the analysis in Section 4 imply that these offers can be chosen such that (i) offer $(\hat{v}^s, \hat{t}^s)$ satisfies the inequalities (17) to (19) and is different from any offer (v_i^s, t_i^s) for $i \in \mathbb{N}$, (ii) given the sender's strategy, it is a PPE for the receiver to accept each offer[14] unless it turns out that $v^o > \bar{v}_i$ after offer (v_i^s, t_i^s) has been made, and (iii) we have $v_i^s + t_i^s < \underline{v}_i$ for each $i \in \mathbb{N}$. For off-equilibrium offers we again assume optimistic beliefs so that they are unprofitable for the sender. We then get that the sender can benefit from making early offers even when there is no asymmetric information between the sender and the receiver. We summarize this result in the following result.

Corollary 2 (Uncertain Outside Option Value)**.** *Consider the model with uncertainty about the outside option value. If F is weakly convex and $\eta(\lambda - 1) > \max\{1 + \eta, 3\}$, an equilibrium exists in which the receiver accepts an inferior offer at each outside option value $v^o > 0$ when the sender knows v^o, and she accepts an offer that is inferior to her outside option in expectation when he does not know v^o.*

6.3 Costly Transfers

So far, we assumed that regular value v^s and transfer t^s are fungible for the sender. It is then attractive for the sender to offer transfers to exploit the attachment effect. In this subsection,

[14] Specifically, we can use arguments very similar to those used in the proof of Lemma 1 to show that the plan "accept $(\hat{v}^s, \hat{t}^s)$ when $v^o \in [0, 1]$" is the receiver's PPE if inequality (18) holds (given that the sender makes this offer only if he has no information about the outside option).

we show that our main result remains unchanged even if exchanging value for transfer is costly for the sender, provided these costs are not too large.

We consider our baseline setting with the following variation. If the sender offers (v^s, t^s) and the receiver accepts this offer, then the sender's payoff is $1 - v^s - (1+\alpha)t^s$ for some parameter value $\alpha \geq 0$. If $\alpha = 0$, the two dimensions are again fungible. If $\alpha > 0$, it is costly for the sender to offer a transfer instead of regular value. In the following, we derive up to which threshold value of α the sender benefits from making early offers. Suppose that $\eta(\lambda-1) > 3$ and that the sender offers (v_i^s, t_i^s) if the receiver's outside option value is in the interval V_i. The analysis in Section 4 shows that it is optimal for the receiver in period 1 to adopt the plan "accept (v_i^s, t_i^s) if $v^o \in V_i$" if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$ and

$$v_i^s + t_i^s > \frac{1}{2}(\bar{v}_i + \underline{v}_i) - \eta(\lambda-1)\frac{1}{6}(\bar{v}_i - \underline{v}_i). \tag{20}$$

Further, if and only if

$$t_i^s = \frac{1+\eta}{1+\eta\lambda}\bar{v}_i - \frac{1+\eta}{1+\eta\lambda}v_i^s, \tag{21}$$

it is optimal for the receiver in period 2 to always accept this offer when $v^o \in V_i$ and to reject it when $v^o > \bar{v}_i$. Recall that the attachment effect captured in this equality enables the sender to reduce the total value $v_i^s + t_i^s$ of the offer by replacing regular value through transfer. Using equation (21) we can show that with costly transfers it is still profitable for the sender to replace regular value through transfer if and only if

$$\alpha < \frac{\eta(\lambda-1)}{1+\eta}. \tag{22}$$

However, this is not the only constraint that needs to be considered. Note that in equilibrium it must be (weakly) optimal for the sender to offer (v_i^s, t_i^s) even if the receiver's outside option is close to the lower bound of V_i. Therefore, we must have

$$v_i^s + (1+\alpha)t_i^s \leq \underline{v}_i. \tag{23}$$

Otherwise, the sender could profitably deviate at outside option values close to $\underline{v}_i$ by making an off-equilibrium offer that has a positive outcome only in the value dimension. Thus, the inequality in (23) must be satisfied for every offer (v_i^s, t_i^s) that the sender makes on the equilibrium path. Using the (in)equalities in (20), (21), and (23), we can derive for which cost parameters α the sender can benefit from making early offers. For given $\bar{v}_i$ and $\underline{v}_i = v_i^s + (1+\alpha)t_i^s$ the constraints in (20) and (21) are satisfied if

$$\alpha < \frac{\eta(\lambda-1)-3}{\eta(\lambda-1)+3} \cdot \frac{\eta(\lambda-1)}{1+\eta}. \tag{24}$$

If this inequality holds, we can construct an equilibrium in which the sender benefits from making early offers at every possible outside option value. For example, it implies that the assessment shown in Figure 1 (where $\eta = 2$ and $\lambda = 3$) is an equilibrium as long as $\alpha < \frac{4}{21}$.

This result can be further extended, e.g., to the setting in Section 5 or to convex cost functions. The latter extension could motivate that the sender offers positive outcomes in both the value and the transfer dimension. The sender-preferred equilibrium then would be shaped by the receiver's loss aversion and the sender's cost function.

6.4 Heterogeneous Degrees of Loss Aversion

Another important assumption that we made in our baseline model is that the sender knows the receiver's degree of loss aversion. We briefly discuss the case when this assumption is no longer satisfied. Suppose there are two loss aversion types of receivers: with probability $\gamma \in (0, 1)$ the receiver exhibits the loss aversion parameters η^*, λ^* with $\eta^*(\lambda^* - 1) > 3$ and with probability $1 - \gamma$ the receiver is loss-neutral. Loss aversion type and outside option value are drawn independently. The rest of the model remains unchanged.

For a loss-neutral receiver type it is rational to accept an offer (v^s, t^s) only if $v^s + t^s \geq v^o$. If the sender wishes that both receiver types accept this offer, it cannot be inferior to the outside option. Thus, the sender faces a trade-off between benefiting from making early offers when the receiver is loss-averse and the probability of acceptance of his offer. Proposition 1 implies that there is an equilibrium in which the sender benefits from making early offers if and only if the probability of a loss-averse receiver γ is sufficiently large.

However, the sender can avoid this trade-off if he is allowed to make an alternative offer after rejection. Suppose that, if the receiver rejects the sender's offer in period 2, then there is a period 3 in which the sender can make another offer $(v^{s,3}, t^{s,3})$ that the receiver can then accept or reject. In this case, the sender can strictly benefit from making early offers, regardless of the share of loss-averse receivers γ. For this, he chooses the early offer (v^s, t^s) according to Proposition 1 for the loss-averse receiver type. The loss-neutral receiver type rejects this offer. In response, the sender chooses the alternative offer $(v^{s,3}, t^{s,3}) = (v^o, 0)$, which the loss-neutral receiver type accepts. Since the alternative offer is equivalent to the outside option, the loss-averse receiver type cannot benefit from also rejecting the early offer (v^s, t^s). Thus, the sender may be able to strictly benefit from making early offers even if the probability of a loss-averse receiver is small.

6.5 Welfare

We briefly comment on the effect of persuasion through early offers on the individual welfare of the sender and the receiver as well as on aggregate welfare. So far, welfare statements are not common in the applied literature on expectation-based loss-averse preferences. The reason for this is that it is typically not clear to what extent gain-loss utility should be treated as part of normative preferences. We follow Goldin and Reck (2022) as well as Reck and Seibold (2023) and introduce a parameter $\pi \in [0,1]$ that captures a social planner's judgment about the normative weight of gain-loss utility. The receiver's welfare in period 2 if she expected to accept the option $(\tilde{v},\tilde{t})$ with certainty and ends up accepting the option (v,t) equals

$$U^*(v,t \mid \tilde{v},\tilde{t}) = v + t + \pi\mu(v-\tilde{v}) + \pi\mu(t-\tilde{t}). \tag{25}$$

Hence, for $\pi = 0$ gain-loss utility is ignored for welfare judgments, while for $\pi = 1$ they receive the same normative weight as consumption utility. The sender's welfare G^* just equals his payoff. For aggregate welfare we use a simple utilitarian welfare function and add up the sender's and receiver's welfare, $G^* + U^*$.

To evaluate the welfare impact of persuasion through early offers, we first determine the equilibrium outcome in the absence of early offers. Suppose the sender can only make offers in period 2 when the receiver also knows her outside option value. As benchmark equilibrium we use the equilibrium in which the sender just matches the receiver's outside option value: At any outside option value v^o, he offers $(v^s,t^s) = (v^o,0)$ in period 2 and the receiver accepts the sender's offer. The expected welfare of the receiver in this equilibrium equals

$$U_0^* = \int_0^1 f(v)v \, \mathrm{d}v - \pi\eta(\lambda-1)\int_0^1 f(v)\int_v^1 f(\tilde{v})(\tilde{v}-v)\,\mathrm{d}\tilde{v}\,\mathrm{d}v, \tag{26}$$

while the expected welfare of the sender is given by

$$G_0^* = \int_0^1 f(v)(1-v)\,\mathrm{d}v. \tag{27}$$

Next, consider a signaling equilibrium in which the sender benefits from making early offers for any outside option value (or a sender-preferred equilibrium) with interval-structure $\{V_i\}_{i\in\mathbb{N}}$ and sender offer (v_i^s,t_i^s) if $v^o \in V_i$. The expected welfare of the receiver in this equilibrium is

$$U^* = \sum_{i=1}^{\infty}\int_{\underline{v}_i}^{\bar{v}_i} f(v)(v_i^s + t_i^s)\,\mathrm{d}v, \tag{28}$$

and the expected welfare of the sender equals

$$G^* = \sum_{i=1}^{\infty} \int_{\underline{v}_i}^{\bar{v}_i} f(v)(1 - v_i^s - t_i^s)\,\mathrm{d}v. \tag{29}$$

From U_0^*, G_0^*, U^*, and G^* we obtain the following results. First, if gain-loss sensations do not matter for welfare judgments, $\pi = 0$, then signaling through early offers has no impact on aggregated welfare. Persuading the receiver to accept an inferior offer only redistributes surplus from the receiver to the sender.

Second, this changes as soon as gain-loss sensations are taken into account for welfare judgments, $\pi > 0$. In this case, signaling through early offers increases aggregated welfare. The reason for this is that, in the considered equilibrium, early offers eliminate all gain-loss sensations in period 2. Formally, the increase in welfare is given by the expected gain-loss sensations in the second term on the right-hand side of equation (26).

Third, for any given value $\pi > 0$, signaling through early offers even implies a Pareto-improvement if $\eta(\lambda - 1)$ is large enough. Observe from equation (26) that U_0^* becomes negative if $\eta(\lambda - 1)$ is large enough, while U^* is strictly positive. Intuitively, this means that the receiver also benefits from obtaining early offers as they eliminate the uncertainty about future outcomes. If this benefit is large enough, both parties are strictly better off from signaling through early offers.

7 Conclusion

In many bargaining settings, parties receive information over time so that initially asymmetric information about possible options becomes symmetric. We showed in this paper that, in this situation, it can be optimal for the better-informed party to make an early offer to an opponent, in particular, if this opponent has reference-dependent loss-averse preferences. The early offer can credibly reveal information and allow the receiver to attain peace of mind at an early stage by planning its acceptance. This allows the sender to persuade the receiver to accept an offer that is inferior to her outside option, even if she has all payoff-relevant information at the decision stage. There would be no such scope for persuasion through signaling if the receiver had standard preferences.

The analysis highlighted several factors for when the sender can persuade the receiver to accept an inferior offer. The offer needs to have features that outside options do not have, so that giving up these features creates loss sensations through the attachment effect. Next, early offers must be made in a way so that some uncertainty about the value of the outside option remains. Through the uncertainty effect it then can be optimal for the receiver to plan acceptance of an offer that with certainty is inferior to her outside option. Therefore, an equilibrium with persuasion through signaling may exhibit an interval-structure akin to that in Crawford and Sobel (1982). The sender-preferred equilibrium optimally balances the strength of the attachment and the uncertainty effect.

Our results of course depend on the point in time when the receiver decides on a plan that determines her reference point. We assumed that the receiver chooses the plan after observing the sender's offer, but before learning the value of her outside option. The scope for persuasion through signaling may be different when the receiver already has a plan (and hence a reference point) in her mind when the sender approaches her with his offer. If the receiver already expects to choose the outside option regardless of its value, then in a signaling equilibrium it may be more difficult (or impossible) to convince her to accept an offer that with certainty has less consumption utility than the outside option. However, if both the sender's offer and the outside option come as a surprise, there may again be scope for persuasion through signaling, depending on how the reference point is adjusted dynamically. Note that if the receiver expects a zero outcome with certainty in both regular value and transfer dimension (before the sender makes an offer), then – according to the personal equilibrium – she has to choose either the offer or the outside option, provided that both opportunities offer a strictly positive outcome in one dimension and a non-negative outcome in the other dimension.

Our setup can be extended in several directions. The literature on persuasion has examined a variety of settings that also could be enriched by taking loss aversion into account, for example, settings with multiple senders, different incentive structures, or incentives to acquire information. The results of the present paper should be helpful for this analysis.

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A Appendix

Proof of Lemma 1. Consider any offer (v^s, t^s) that the sender makes if and only if $v_o \in V \subset (0, 1)$, with $v^s + t^s \leq \underline{v}$. Let $\hat{F}$ be the updated distribution over outside option values when the receiver observes (v^s, t^s). Consider a cut-off plan σ^r where for some $v^* \in [\underline{v}, \bar{v}]$ the receiver accepts (v^s, t^s) if $v^o \in (\underline{v}, v^*]$ and rejects (v^s, t^s) if $[v^*, \bar{v})$. After observing (v^s, t^s), the receiver's expected utility from σ^r equals

$$\begin{aligned}\mathbb{E}_{\hat{F}}[U_R(\sigma^r(v^o, v^s, t^s) \mid G^v, G^t)] \quad = \quad & \hat{F}(v^*)(v^s + t^s) + \int_{v^*}^{\bar{v}} \hat{f}(v)v \, \mathrm{d}v \\ & -\eta(\lambda - 1)\hat{F}(v^*)[1 - \hat{F}(v^*)]t^s \\ & -\eta(\lambda - 1)\hat{F}(v^*) \int_{v^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s) \, \mathrm{d}\tilde{v} \\ & -\eta(\lambda - 1) \int_{v^*}^{\bar{v}} \hat{f}(v) \int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v. \qquad (30)\end{aligned}$$

We show that this term is maximal either at $v^* = \underline{v}$ or at $v^* = \bar{v}$ or at both points. For this, we differentiate receiver's expected utility with respect to v^*:

$$\begin{aligned}\frac{\partial \mathbb{E}_{\hat{F}}[\cdot]}{\partial v^*} \quad = \quad & \hat{f}(v^*)(v^s + t^s) - \hat{f}(v^*)v^* \\ & -\eta(\lambda - 1)\hat{f}(v^*)[1 - 2\hat{F}(v^*)]t^s \\ & -\eta(\lambda - 1)\left[\hat{f}(v^*) \int_{v^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s) \, \mathrm{d}\tilde{v} - \hat{F}(v^*)\hat{f}(v^*)(v^* - v^s)\right] \\ & +\eta(\lambda - 1)\hat{f}(v^*) \int_{v^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^*) \, \mathrm{d}\tilde{v}. \qquad (31)\end{aligned}$$

We can simplify this to

$$\frac{\partial \mathbb{E}_{\hat{F}}[\cdot]}{\partial v^*} = -\hat{f}(v^*)\left[(v^* - v^s - t^s) + \eta(\lambda - 1)(1 - 2\hat{F}(v^*))(v^* - v^s + t^s)\right]. \qquad (32)$$

Since $v^s + t^s \leq \underline{v} \leq v^*$, this term is strictly negative for all $v^* > \underline{v}$ with $\hat{F}(v^*) \leq \frac{1}{2}$. Denote by $\Gamma(v^*)$ the term in the squared brackets in (32). The derivative $\frac{\partial \mathbb{E}_{\hat{F}}[\cdot]}{\partial v^*}$ is positive (negative) if and only if $\Gamma(v^*)$ is negative (positive). Consider the derivative

$$\frac{\partial \Gamma(v^*)}{\partial v^*} = 1 + \eta(\lambda - 1)[-2\hat{f}(v^*)(v^* - v^s + t^s) + (1 - 2\hat{F}(v^*))]. \qquad (33)$$

Since F is weakly convex, $\hat{f}$ weakly increases in v^* on its support. Hence, the right-hand side of equation (33) strictly decreases in v^*. If $\frac{\partial \Gamma(v^*)}{\partial v^*}$ is negative at $v^* = v^{**}$, it is negative for all $v^* > v^{**}$. By the statement above, if $\frac{\partial \mathbb{E}_{\hat{F}}[\cdot]}{\partial v^*}$ becomes positive at some value $v^* = v^{**}$, it remains positive for all $v^* > v^{**}$, which yields us the result. $\square$

Proof of Proposition 1. The proof proceeds by steps. **Step 1.** Consider an interval $V_i = (\underline{v}_i, \bar{v}_i] \subset (0, 1]$ and assume that the sender makes the offer (v_i^s, t_i^s) to the receiver if and only if $v^o \in V_i$. We show that if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, then we can choose (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$ such that the receiver's PPE specifies to accept this offer whenever $v^o \in V_i$ and to reject it when $v^o > \bar{v}_i$. Lemma 1 implies that the plan "accept (v_i^s, t_i^s) if $v^o \in V_i$" is the payoff-maximizing plan for the receiver if its expected payoff exceeds that from the plan "accept the outside option if $v^o \in V_i$." Her expected utility from the latter plan equals

$$\int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) v \, \mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v. \tag{34}$$

Note that $\hat{f}(v) = \frac{1}{F(\bar{v}_i) - F(\underline{v}_i)} f(v)$. We assumed that $0 < f(v^o) < \infty$ for all $v^o \in [0, 1]$. Hence, we have $\frac{\hat{f}(\bar{v}_i) - \hat{f}(\underline{v}_i)}{\hat{f}(\underline{v}_i)} \to 0$ for $\underline{v}_i \to \bar{v}_i$. Therefore, the distribution $\hat{F}(v)$ becomes the uniform distribution with density $\frac{1}{\bar{v}_i - \underline{v}_i}$ as $\underline{v}_i$ get close to $\bar{v}_i$. The expected utility from always choosing the outside option then becomes

$$\frac{1}{\bar{v}_i - \underline{v}_i} \int_{\underline{v}_i}^{\bar{v}_i} 1 \, \mathrm{d}v - \eta(\lambda - 1) \frac{1}{(\bar{v}_i - \underline{v}_i)^2} \int_{\underline{v}_i}^{\bar{v}_i} \int_{v}^{\bar{v}_i} (\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v, \tag{35}$$

which we can simplify to

$$\frac{1}{2}(\bar{v}_i + \underline{v}_i) - \eta(\lambda - 1)\frac{1}{6}(\bar{v}_i - \underline{v}_i). \tag{36}$$

The expected utility from accepting the sender's offer equals $v_i^s + t_i^s$. Note that

$$\underline{v}_i > \frac{1}{2}(\bar{v}_i + \underline{v}_i) - \eta(\lambda - 1)\frac{1}{6}(\bar{v}_i - \underline{v}_i) \tag{37}$$

is equivalent to $\eta(\lambda - 1) > 3$. Hence, if $\eta(\lambda - 1) > 3$ and $\underline{v}_i$ is close enough to $\bar{v}_i$, we can find values (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$ such that, in period 1, the plan "accept (v_i^s, t_i^s) if $v^o \in V_i$" is preferred to any other plan. Next, we check when it is optimal for the receiver to also execute this plan in period 2 (so that it is indeed a PPE). Her utility from following the plan is $v_i^s + t_i^s$, while her utility from choosing the outside option equals $v^o + \eta(v^o - v_i^s) - \eta\lambda t_i^s$. If $\underline{v}_i > \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$ we can choose offer (v_i^s, t_i^s) with $t_i^s > 0$ and $v_i^s + t_i^s < \underline{v}_i$ such that

$$(1 + \eta)v_i^s + (1 + \eta\lambda)t_i^s = (1 + \eta)\bar{v}_i. \tag{38}$$

It is then optimal for the receiver to accept offer (v_i^s, t_i^s) if $v^o \in V_i$ and to reject it if $v^o > \bar{v}_i$, which completes the proof of the statement. **Step 2**. We construct a signaling equilibrium with the desired property. We can choose a sequence of half-open intervals $\{V_i\}_{i \in \mathbb{N}}$ with $\underline{v}_i > \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$ for each $i \in \mathbb{N}$ and a sequence of offers $\{(v_i^s, t_i^s)\}_{i \in \mathbb{N}}$ so that the sender makes the offer (v_i^s, t_i^s)

with total value $0 < v_i^s + t_i^s < \underline{v}_i$ to the receiver whenever $v^o \in V_i$ and $v_i^s + t_i^s$ strictly decreases in i. By Step 1, this sequence can be chosen so that it is a PPE for the receiver to accept the sender's offer (v_i^s, t_i^s) if $v^o \leq \bar{v}_i$ and to reject it otherwise. For any offer (v^s, t^s) that is not an element of the set $\{(v_i^s, t_i^s)\}_{i\in\mathbb{N}}$ we specify that in period 1 the receiver believes that her outside option value is $v^o = 1$ with certainty. It is then optimal for her to reject an offer (v^s, t^s) in period 2 if $v^s + t^s \leq v^o$. Given this receiver behavior, it is then indeed optimal for the sender to offer (v_i^s, t_i^s) if and only if $v^o \in V_i$. This completes the proof of the first statement of Proposition 1. **Step 3**. We prove the second statement of Proposition 1. Note that in equilibrium it must be the case that the receiver always accepts the sender's offer. Hence, for any two values $v, \hat{v} \in [0, 1]$ with $v > \hat{v}$ the following must hold: Suppose the sender offers the total value w if $v^o = v$ and $\hat{w}$ if $v^o = \hat{v}$. Then we must have $w \geq \hat{w}$. Otherwise, the sender could deviate profitably if $v^o = \hat{v}$ by making the same offer as for $v^o = v$ since the receiver will accept it. Given this result, we can make the following observation: Assume by contradiction that there exists an interval $V = (v_L, v_H) \subset [0, 1]$ so that for any two outside option values $v, \hat{v} \in V$ the sender makes offers with varying total value, $w \neq \hat{w}$. The receiver would then be able to infer her outside option value from these offers so that the sender cannot persuade her accept an inferior offer, a contradiction. Hence, an equilibrium in which the sender always benefits from making early offers must be characterized by a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$, so that the sender makes an offer with total value $v_i^s + t_i^s < \underline{v}_i$ and $t_i > 0$ if $v^o \in V_i$. **Step 4**. We show that if $\eta(\lambda-1) < 3$, there exists no equilibrium in which the sender benefits from making early offers. Note that we can rewrite

$$\int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v)\, \mathrm{d}\tilde{v}\, \mathrm{d}v = \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{\underline{v}_i}^{v} \hat{f}(\tilde{v})(v - \tilde{v})\, \mathrm{d}\tilde{v}\, \mathrm{d}v. \tag{39}$$

Moreover, we have

$$\begin{aligned} & \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v)\, \mathrm{d}\tilde{v}\, \mathrm{d}v + \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{\underline{v}_i}^{v} \hat{f}(\tilde{v})(v - \tilde{v})\, \mathrm{d}\tilde{v}\, \mathrm{d}v \\ = & \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \left[\int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v)\, \mathrm{d}\tilde{v} + \int_{\underline{v}_i}^{v} \hat{f}(\tilde{v})(v - \tilde{v})\, \mathrm{d}\tilde{v} \right] \mathrm{d}v \\ = & \int_{\underline{v}_i}^{\bar{v}_i} \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v)\hat{f}(\tilde{v}) \mid \tilde{v} - v \mid \mathrm{d}\tilde{v}\, \mathrm{d}v. \end{aligned} \tag{40}$$

We therefore can write the expected payoff from the plan "accept the outside option if $v^o \in V_i$" in (34) as

$$\int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v)\, \mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}_i}^{\bar{v}_i} \int_{\underline{v}_i}^{\bar{v}_i} \frac{\hat{f}(v)\hat{f}(\tilde{v})}{2} \mid v - \tilde{v} \mid \mathrm{d}\tilde{v}\, \mathrm{d}v. \tag{41}$$

Observe from (41) that the receiver's expected payoff from accepting the outside option is minimal if $\hat{F}$ is a uniform distribution provided that $\hat{f}$ is weakly increasing on its support. To see this, note that the unique maximum of the product $\hat{f}(v)\hat{f}(\tilde{v})$ subject to the constraint that $\hat{f}(v) + \hat{f}(\tilde{v}) = G$ for some value G is obtained when $\hat{f}(v) = \hat{f}(\tilde{v}) = \frac{G}{2}$. Hence, by Step 1, if the support of $\hat{F}$ is an interval V, the receiver accepts an offer with total value $v^s + t^s < \underline{v}_i$ only if $\eta(\lambda - 1) \geq 3$, which completes the proof. □

Proof of Proposition 2. The proof proceeds in steps. **Step 1.** We prove the first statement. In an equilibrium, the receiver always accepts the sender's offer if $v^o < 1$. As in Step 3 of the proof of Proposition 1 we can show that if the sender offers the total value w if $v^o = v < 1$ and $\hat{w}$ if $v^o = \hat{v} < v$, then we must have $w \geq \hat{w}$. Assume by contradiction that in a sender-preferred equilibrium σ exists an interval $V = (v_L, v_H) \subset [0, 1]$ so that for any two outside option values $v, \hat{v} \in V$ the sender makes offers with varying total value, $w \neq \hat{w}$. The receiver would then be able to infer her outside option value from these offers so that the total value of a sender offer equals the outside option value for each $v^o \in V$. We then can find an alternative equilibrium σ' that is identical to σ except that there is an interval of outside option values $(v'_L, v'_H) \subset (v_L, v_H)$ at which the sender makes an offer with total value $w < v_L$. This can be shown by following the same steps as in Step 1 of the proof of Proposition 1. The sender's expected payoff in equilibrium σ' strictly exceeds that in equilibrium σ, a contradiction. This implies the first statement. **Step 2.** We prove the second statement. Consider any sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$ with the property that, for each interval V_i, we have $\underline{v}_i = \bar{v}_{i+1}$ and $\frac{1+\eta}{1+\eta\lambda}\bar{v}_i \leq \underline{v}_i \leq \Gamma(\eta, \lambda)\bar{v}_i$. By Step 1, a sender-preferred equilibrium must be characterized by such a sequence. In such an equilibrium, the sender offers (v_i^s, t_i^s) if $v^o \in V_i$, and the receiver accepts this offer. Define $w_i^s = v_i^s + t_i^s$. It is optimal for the receiver to plan acceptance in period 1 if and only if

$$w_i^s \geq \frac{1}{2}(\bar{v}_i + \underline{v}_i) - \frac{1}{6}\eta(\lambda - 1)(\bar{v}_i - \underline{v}_i) \tag{42}$$

for all $i \in \mathbb{N}$. For actual acceptance in period 2, it must be the case that condition (7) is satisfied. Hence, we must have

$$w_i^s \geq \frac{1+\eta}{1+\eta\lambda}\bar{v}_i. \tag{43}$$

Note that this inequality is satisfied with equality if and only if $(v_i^s, t_i^s) = (0, \frac{1+\eta}{1+\eta\lambda}\bar{v}_i)$. The inequality in (42) defines an upper bound on $\underline{v}_i$:

$$\underline{v}_i \leq \frac{1}{\frac{1}{2} + \frac{1}{6}\eta(\lambda - 1)}\left[w_i^s + \bar{v}_i\left(\frac{1}{6}\eta(\lambda - 1) - \frac{1}{2}\right)\right]. \tag{44}$$

Consider the average total value offered as a fraction of the average outside option value in interval V_i. It is given by

$$av_i = \frac{w_i^s}{\frac{1}{2}(\bar{v}_i + \underline{v}_i)}. \tag{45}$$

We show that for given $\bar{v}_i$ the lowest possible equilibrium value of av_i is obtained when the inequalities in (43) and (44) are satisfied with equality, and that this value only depends on η and λ. Observe that av_i strictly decreases in $\underline{v}_i$. The largest possible equilibrium value of $\underline{v}_i$ is given by the right-hand side of inequality (44). We replace $\underline{v}_i$ by the right-hand side of inequality (44) in equation (45) and obtain

$$av_i = \frac{(1 + \frac{1}{3}\eta(\lambda - 1))w_i^s}{\bar{v}_i\frac{1}{3}\eta(\lambda - 1) + w_i^s}. \tag{46}$$

This expression strictly increases in w_i^s. Therefore, the lowest possible equilibrium value of av_i is obtained if $w_i^s = \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$. If we further replace w_i^s by $\frac{1+\eta}{1+\eta\lambda}\bar{v}_i$, the term $\bar{v}_i$ drops out of the resulting expression, which proves the claim. From this, the second statement of Proposition 2 directly follows. □

Proof of Lemma 2. Consider an offer (v^s, t^s) that the sender makes if and only if $v_o \in V \subset (0, 1)$, with $v^s + t^s \leq v^o$. Let $\hat{F}$ be the updated distribution over outside option values when the receiver observes (v^s, t^s). Since F is weakly convex, $\hat{f}$ weakly increases on its support. Consider w.l.o.g. a cut-off plan σ^r characterized by two values $v_1^*, v_2^* \in [\underline{v}, \bar{v}]$ with $v_1^* \leq v_2^*$ that has the following features: In state 1, the receiver accepts (v^s, t^s) if $v^o \in [\underline{v}, v_1^*]$ and rejects (v^s, t^s) if $(v_1^*, \bar{v}]$. In state 2, the receiver accepts (v^s, t^s) if $v^o \in [\underline{v}, v_2^*]$ and rejects (v^s, t^s) if $(v_2^*, \bar{v}]$. After observing (v^s, t^s), the receiver's expected utility from σ^r equals

$$\begin{aligned}
\mathbb{E}_{\hat{F}}[U_R(.)] \quad = \quad & \left(\frac{1}{2}\hat{F}(v_1^*) + \frac{1}{2}\hat{F}(v_2^*)\right)(v^s + t^s) + \frac{1}{2}\int_{v_1^*}^{\bar{v}} \hat{f}(v)v \,\mathrm{d}v + \frac{1}{2}\int_{v_2^*}^{\bar{v}} \hat{f}(v)v \,\mathrm{d}v \\
& -\eta(\lambda - 1)\left(\frac{1}{2}\hat{F}(v_1^*) + \frac{1}{2}\hat{F}(v_2^*)\right)\left(1 - \frac{1}{2}\hat{F}(v_1^*) - \frac{1}{2}\hat{F}(v_2^*)\right)t^s \\
& -\eta(\lambda - 1)\left(\frac{1}{2}\hat{F}(v_1^*) + \frac{1}{2}\hat{F}(v_2^*)\right)\left(\frac{1}{2}\int_{v_1^*}^{v_2^*} \hat{f}(\tilde{v})(\tilde{v} - v^s)\,\mathrm{d}\tilde{v} + \int_{v_2^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s)\,\mathrm{d}\tilde{v}\right) \\
& -\eta(\lambda - 1)\frac{1}{2}\int_{v_1^*}^{v_2^*} \hat{f}(v)\left(\frac{1}{2}\int_{v}^{v_2^*} \hat{f}(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v} + \int_{v_2^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v}\right)\mathrm{d}v \\
& -\eta(\lambda - 1)\int_{v_2^*}^{\bar{v}} \hat{f}(v)\int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v}\,\mathrm{d}v \\
& -\eta(\lambda - 1)\left(1 - \frac{1}{2}\hat{F}(v_1^*)^2 - \frac{1}{2}\hat{F}(v_2^*)^2\right)\xi.
\end{aligned} \tag{47}$$

The rest of the proof proceeds in steps. **Step 1.** We show that for given $v_2^* > \underline{v}$ the value in (47) is maximal at $v_1^* = \underline{v}$ or at $v_1^* = v_2^*$ or at both values. The first derivative of (47) with respect to

v_1^* equals

$$\begin{aligned}
\frac{\partial \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial v_1^*} &= -\frac{1}{2}\hat{f}(v_1^*)(v_1^* - v^s - t^s) \\
&\quad -\eta(\lambda-1)\frac{1}{2}\hat{f}(v_1^*)(1-\hat{F}(v_1^*)-\hat{F}(v_2^*))t^s \\
&\quad +\eta(\lambda-1)\frac{1}{2}\hat{f}(v_1^*)\left(\frac{1}{2}\hat{F}(v_1^*)+\frac{1}{2}\hat{F}(v_2^*)\right)(v_1^*-v^s) \\
&\quad -\eta(\lambda-1)\frac{1}{2}\hat{f}(v_1^*)\left(\frac{1}{2}\int_{v_1^*}^{v_2^*}\hat{f}(\tilde{v})(\tilde{v}-v^s)\,\mathrm{d}\tilde{v}+\int_{v_2^*}^{\bar{v}}\hat{f}(\tilde{v})(\tilde{v}-v^s)\,\mathrm{d}\tilde{v}\right) \\
&\quad +\eta(\lambda-1)\frac{1}{2}\hat{f}(v_1^*)\left(\frac{1}{2}\int_{v_1^*}^{v_2^*}\hat{f}(\tilde{v})(\tilde{v}-v_1^*)\,\mathrm{d}\tilde{v}+\int_{v_2^*}^{\bar{v}}\hat{f}(\tilde{v})(\tilde{v}-v_1^*)\,\mathrm{d}\tilde{v}\right) \\
&\quad +\eta(\lambda-1)\hat{f}(v_1^*)\hat{F}(v_1^*)\xi, \qquad (48)
\end{aligned}$$

which can be simplified to

$$\begin{aligned}
\frac{\partial \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial v_1^*} &= -\frac{1}{2}\hat{f}(v_1^*)\Bigg[(v_1^*-v^s-t^s)+\eta(\lambda-1) \\
&\quad \times\left((1-\hat{F}(v_1^*)-\hat{F}(v_2^*))t^s+\left(1-\frac{3}{2}\hat{F}(v_1^*)-\frac{1}{2}\hat{F}(v_2^*)\right)(v_1^*-v^s)-2\hat{F}(v_1^*)\xi\right)\Bigg]. \qquad (49)
\end{aligned}$$

Denote the term in squared brackets by $\Gamma_1(v_1^*, v_2^*)$. The second derivative of (47) with respect to v_1^* equals

$$\begin{aligned}
\frac{\partial^2 \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial (v_1^*)^2} &= -\frac{1}{2}\hat{f}'(v_1^*)\Gamma_1(v_1^*,v_2^*)-\frac{1}{2}\hat{f}(v_1^*)\Bigg[1+\eta(\lambda-1) \\
&\quad \times\left(-\hat{f}(v_1^*)\left(\frac{3}{2}(v_1^*-v^s)+t^s+2\xi\right)+\left(1-\frac{3}{2}\hat{F}(v_1^*)-\frac{1}{2}\hat{F}(v_2^*)\right)\right)\Bigg]. \qquad (50)
\end{aligned}$$

Assume by contradiction that $\mathbb{E}_{\hat{F}}[U_R(.)]$ has a local maximum at $\hat{v}_1^* \in (\underline{v}, v_2^*)$. Note that the first derivative of $\mathbb{E}_{\hat{F}}[U_R(.)]$ with respect to v_1^* is strictly negative at $v_1^* = \underline{v}$. Hence, there must be a local minimum of $\mathbb{E}_{\hat{F}}[U_R(.)]$ at some value $\tilde{v}_1^* \in (\underline{v}, \hat{v}_1^*)$. At a local maximum or minimum, we must have $\Gamma_1(., v_2^*) = 0$. Therefore, the term in squared brackets on the right-hand side of equation (50) must be negative at $v_1^* = \tilde{v}_1^*$ and positive at $v_1^* = \hat{v}_1^*$. This implies that

$$\hat{f}(\hat{v}_1^*)\left(\frac{3}{2}(\hat{v}_1^*-v^s)+t^s+2\xi\right)+\frac{3}{2}\hat{F}(\hat{v}_1^*) < \hat{f}(\tilde{v}_1^*)\left(\frac{3}{2}(\tilde{v}_1^*-v^s)+t^s+2\xi\right)+\frac{3}{2}\hat{F}(\tilde{v}_1^*), \qquad (51)$$

which contradicts the fact that $\hat{f}$ weakly increases on its support and $\hat{v}_1^* > \tilde{v}_1^*$. This completes the proof of the statement. **Step 2.** We show that at $v_1^* = \underline{v}$ the expected payoff in (47) is maximal at $v_2^* = \underline{v}$ or at $v_2^* = \bar{v}$ or at both values. The first derivative of (47) with respect to v_2^* equals

$$\begin{aligned}\frac{\partial \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial v_2^*} \quad = \quad & -\frac{1}{2}\hat{f}(v_2^*)\Bigg[(v_2^* - v^s - t^s) + \eta(\lambda-1)\Bigg((1-\hat{F}(v_1^*) - \hat{F}(v_2^*))t^s \\ & + \left(1 - \frac{1}{2}\hat{F}(v_1^*) - \frac{3}{2}\hat{F}(v_2^*)\right)(v_2^* - v^s) + \frac{1}{2}\int_{v_1^*}^{v_2^*} \hat{f}(\tilde{v})(2\tilde{v} - v^s - v_2^*)\,\mathrm{d}\tilde{v} - 2\hat{F}(v_2^*)\xi\Bigg)\Bigg].\end{aligned} \tag{52}$$

Denote the term in squared brackets by $\Gamma_2(v_1^*, v_2^*)$. The second derivative of (47) with respect to v_2^* equals

$$\begin{aligned}\frac{\partial^2 \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial (v_2^*)^2} \quad = \quad & -\frac{1}{2}\hat{f}'(v_2^*)\Gamma_2(v_1^*, v_2^*) - \frac{1}{2}\hat{f}(v_2^*)\Bigg[1 + \eta(\lambda-1) \\ & \times\left(-\hat{f}(v_2^*)\,(v_2^* - v^s + t^s + 2\xi) + \left(1 - 2\hat{F}(v_2^*)\right)\right)\Bigg].\end{aligned} \tag{53}$$

Note that, at $v_1^* = \underline{v}$, the first derivative of $\mathbb{E}_{\hat{F}}[U_R(.)]$ with respect to v_2^* is strictly negative. By applying the same arguments as in Step 1, we then can show the result. **Step 3.** We consider the set of cut-off plans with $v_1^* = v_2^* = v^*$ and show that the expected payoff in (47) for these plans is maximal at $v^* = \underline{v}$ or at $v^* = \bar{v}$ or at both values. The first derivative of (47) with respect to v^* is

$$\frac{\partial \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial v^*} = -\hat{f}(v^*)\left[(v_1^* - v^s - t^s) + \eta(\lambda-1)\left((1-\hat{F}(v^*))(v_1^* - v^s + t^s) - 2\hat{F}(v^*)\xi\right)\right] \tag{54}$$

and the second derivative of (47) with respect to v^* equals

$$\begin{aligned}\frac{\partial^2 \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial (v^*)^2} \quad = \quad & -\hat{f}'(v^*)\Gamma_3(v^*) - \hat{f}(v^*)\Bigg[1 + \eta(\lambda-1) \\ & \times\left(-2\hat{f}(v^*)(v^* - v^s + t^s + 2\xi) + (1 - 2\hat{F}(v^*))\right)\Bigg],\end{aligned} \tag{55}$$

where $\Gamma_3(v^*)$ is the term in squared brackets on the right-hand side of equation (54). We can now apply the same arguments as in Step 1 and Step 2 to prove the statement. **Step 4.** From Steps 1 to 3 it follows that only the following three cut-off plans potentially maximize the expected payoff in equation (47): a plan with $v_1^* = v_2^* = \bar{v}$, a plan with $v_1^* = v_2^* = \underline{v}$, and a plan with $v_1^* = \underline{v}$ and $v_2^* = \bar{v}$. We show that the expected payoff from the last plan is always strictly smaller than the expected payoff of the first or of the second plan. The expected payoff from the first plan is $U_1 = v^s + t^s$. The expected payoff of the second plan is

$$U_2 = \int_{\underline{v}}^{\bar{v}} \hat{f}(v) v \,\mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}}^{\bar{v}} \hat{f}(v) \int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v) \,\mathrm{d}\tilde{v} \,\mathrm{d}v - \eta(\lambda - 1)\xi, \tag{56}$$

and the expected payoff from the third plan is

$$\begin{aligned} U_3 \;&=\; \frac{1}{2}(v^s + t^s) + \frac{1}{2}\int_{\underline{v}}^{\bar{v}} \hat{f}(v) v \,\mathrm{d}v - \eta(\lambda - 1)\frac{1}{4}t^s - \eta(\lambda - 1)\frac{1}{4}\int_{\underline{v}}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s) \,\mathrm{d}\tilde{v} + \\ &\quad -\eta(\lambda - 1)\frac{1}{4}\int_{\underline{v}}^{\bar{v}} \hat{f}(v) \int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v) \,\mathrm{d}\tilde{v} \,\mathrm{d}v - \eta(\lambda - 1)\frac{1}{2}\xi. \end{aligned} \tag{57}$$

Note that

$$\int_{\underline{v}}^{\bar{v}} \hat{f}(v) \int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v) \,\mathrm{d}\tilde{v} \,\mathrm{d}v < \int_{\underline{v}}^{\bar{v}} \hat{f}(v) \int_{\underline{v}}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s) \,\mathrm{d}\tilde{v} \,\mathrm{d}v = \int_{\underline{v}}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s) \,\mathrm{d}\tilde{v}. \tag{58}$$

We can use this to show that if $U_1 \geq U_2$, then we also have $U_1 > U_3$; and if $U_2 \geq U_1$, then we also have $U_2 > U_3$, which completes the proof. □

Proof of Proposition 3. The proof proceeds in two steps. **Step 1.** Consider an interval $V_i = (\underline{v}_i, \bar{v}_i] \subset (\frac{\eta(\lambda-1)}{1+\eta}\xi, 1]$ and assume that the sender makes the offer (v_i^s, t_i^s) to the receiver if and only if $v^o \in V_i$. We show that if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, then we can choose (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$ such that the receiver's PPE specifies to accept this offer whenever $v^o \in V_i$. Lemma 2 implies that the plan "accept (v_i^s, t_i^s) if $v^o \in V_i$" is the payoff-maximizing plan for the receiver if its expected payoff exceeds that from the plan "accept the outside option if $v^o \in V_i$." Her expected payoff from the latter plan after observing offer (v_i^s, t_i^s) equals

$$\int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) v \,\mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v) \,\mathrm{d}\tilde{v} \,\mathrm{d}v - \eta(\lambda - 1)\xi. \tag{59}$$

As in the proof of Proposition 1, we can show that for $\underline{v}_i \to \bar{v}_i$ this expression becomes

$$\frac{1}{2}(\bar{v}_i + \underline{v}_i) - \eta(\lambda - 1)\frac{1}{6}(\bar{v}_i - \underline{v}_i) - \eta(\lambda - 1)\xi. \tag{60}$$

By assumption, we have $\eta(\lambda - 1) > 0$. Hence, if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, then we can find values (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$ such that the payoff-maximizing plan in period 1 is to accept (v_i^s, t_i^s) if $v^o \in V_i$. We examine when this plan is consistent with a PPE. In period 2, the receiver's payoff from accepting the sender's offer is $v^s + t^s$, while the payoff from accepting the outside option value is, in both states, equal to

$$v^o + \eta(v^o - v^s) - \eta\lambda t^s - \eta(\lambda - 1)\xi. \tag{61}$$

The receiver is indifferent between the sender's offer and the outside option at $v^o = \bar{v}_i$ if

$$\bar{v}_i + \eta(\bar{v}_i - v^s) - \eta\lambda t^s - \eta(\lambda - 1)\xi = v_i^s + t_i^s. \tag{62}$$

We can find values (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$ that satisfy this equality if

$$\bar{v}_i + \eta\bar{v}_i - \eta\lambda\underline{v}_i - \eta(\lambda - 1)\xi < \underline{v}_i, \tag{63}$$

which is equivalent to

$$\frac{1+\eta}{1+\eta\lambda}\bar{v}_i - \frac{\eta(\lambda-1)}{1+\eta\lambda}\xi < \underline{v}_i. \tag{64}$$

Hence, if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, we can find values (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$ such that in the PPE the receiver always accepts (v_i^s, t_i^s) if $v^o \in V_i$. **Step 2.** We can now construct the desired equilibrium. Suppose the sender adopts the following strategy: If $v^o \in [0, \frac{\eta(\lambda-1)}{1+\eta}\xi]$, the sender offers $(v^s, t^s) = (0, 0)$ to the receiver. The interval $(\frac{\eta(\lambda-1)}{1+\eta}\xi, 1]$ is partitioned by a finite sequence of disjoint half-open intervals $\{V_i\}_{i=1,...,n}$ so that the sender offers (v_i^s, t_i^s) to the receiver if $v^o \in V_i$. For each $i = 1, ..., n$, the interval V_i as well as the values v_i^s, t_i^s are chosen such that the receiver is indifferent between the sender's offer and the outside option at $v^o = \bar{v}_i$ in period 2 and accepting on-equilibrium offers characterizes the receiver's PPE. In Step 1, we have shown that this is possible. Assuming optimistic beliefs for off-equilibrium offers then ensures that no party can deviate profitably. □

Proof of Proposition 4. Assume that there is a sender-preferred equilibrium σ in which, for any value $\underline{v} \in (0, 1)$, there is no bunching of the total value $v^s + t^s$ at the outside option values in $(\underline{v}, 1]$. We modify σ so that we obtain an equilibrium with bunching at the highest outside option values, which dominates σ in terms of expected payoff for the sender. The assumption on σ implies that there is a $v^* \in (0, 1)$ so that for each $v^o > v^*$ the sender's offers signal the precise outside option value to the receiver. The expected total value that the sender offers to the receiver in equilibrium σ given that $v^o > v^*$ is at least

$$\int_{v^*}^1 v\hat{f}(v)dv - \eta(\lambda - 1)\xi, \tag{65}$$

where $\hat{f}$ is the density conditional on $v^o > v^*$. We find a value $\underline{v}$ so that the following inequalities hold:

$$\underline{v} < \int_{\underline{v}}^{1} v\hat{f}(v)dv - \eta(\lambda-1)\xi, \tag{66}$$

$$\underline{v} \geq \int_{\underline{v}}^{1} v\hat{f}(v)dv - \eta(\lambda-1)\int_{\underline{v}}^{1}\hat{f}(v)\int_{v}^{1}\hat{f}(\tilde{v})(\tilde{v}-v)d\tilde{v}dv - \eta(\lambda-1)\xi, \tag{67}$$

and

$$\underline{v} \geq \frac{1+\eta}{1+\eta\lambda} - \frac{\eta(\lambda-1)\xi}{1+\eta\lambda}. \tag{68}$$

Observe that, if ξ is small enough, then such a value $\underline{v}$ exists. We now modify σ as follows. At all outside option values in the interval $(\underline{v}, 1]$ the sender makes an offer (v^s, t^s) with total value $v^s + t^s = \underline{v}$ and sufficiently large value t^s; otherwise, the sender's strategy remains the same. The three inequalities above ensure that in a PPE the receiver always accepts (v^s, t^s) when $v^o \in (\underline{v}, 1]$, and that the sender's expected payoff under the modified equilibrium is strictly larger than under the original equilibrium. □

Proof of Proposition 5. The proof proceeds by steps. In Steps 1 to 3, we show that the assessment stated in Proposition 5 is an equilibrium outcome. In Step 4, we show that any sender-preferred equilibrium is consistent with this assessment. **Step 1.** We find the interval-structure $\{V_i\}_{i\in\mathbb{N}}$ and offers (v_i^s, t_i^s) for each $i \in \mathbb{N}$ so that the expected total value offered in equilibrium is minimal for outside option values in the interval $(\eta\lambda\xi, 1]$. For convenience, we abbreviate $w_i^s = v_i^s + t_i^s$. It is optimal for the receiver to plan acceptance in period 1 if and only if

$$w_i^s \geq \frac{1}{2}(\bar{v}_i + \underline{v}_i) - \frac{1}{6}\eta(\lambda-1)(\bar{v}_i - \underline{v}_i) - \eta(\lambda-1) \tag{69}$$

for all $i \in \mathbb{N}$. The lowest total value that the sender must offer so that the receiver actually accepts (v_i^s, t_i^s) if $v^o = \bar{v}_i$ is defined by

$$t_i^s = (1+\eta)\bar{v}_i - \eta\lambda t_i^s - \eta(\lambda-1)\xi. \tag{70}$$

We therefore set

$$w_i^s = \frac{(1+\eta)\bar{v}_i - \eta(\lambda-1)\xi}{1+\eta\lambda}. \tag{71}$$

The inequality in (69) defines an upper bound on $\underline{v}_i$:

$$\underline{v}_i \leq \frac{1}{\frac{1}{2} + \frac{1}{6}\eta(\lambda-1)}\left[w_i^s + \bar{v}_i\left(\frac{1}{6}\eta(\lambda-1) - \frac{1}{2}\right) + \eta(\lambda-1)\xi\right]. \tag{72}$$

We consider the average total value offered as a fraction of the average outside option value in interval V_i. It is given by

$$av_i = \frac{w_i^s}{\frac{1}{2}(\bar{v}_i + \underline{v}_i)}. \tag{73}$$

As in Step 2 of the proof of Proposition 2, we can now show that for given $\bar{v}_i$ the lowest possible equilibrium value of av_i is obtained when the inequalities in (71) and (72) are satisfied with equality. Importantly, we can show that if $\bar{v}_i > \eta\lambda\xi$, then the value of $\underline{v}_i$ defined by (72) satisfies $\underline{v}_i < \bar{v}_i$ and $\underline{v}_i > \eta\lambda\xi$ so that we indeed obtain an infinite sequence of intervals that partition the interval $(\eta\lambda\xi, 1]$. **Step 2.** For each outside option value $\frac{\eta(\lambda-1)}{1+\eta}\xi < v^o \leq \eta\lambda\xi$ we find the offer (v^s, t^s) that minimizes the total value and that the receiver accepts in equilibrium. At given outside option value v^o the smallest total value that the sender needs to offer so that the receiver accepts the early offer in period 2 is defined by

$$t^s = (1+\eta)v^o - \eta\lambda t^s - \eta(\lambda - 1)\xi. \tag{74}$$

The corresponding offer is $(0, t^s)$ with

$$t^s = \frac{(1+\eta)v^o - \eta(\lambda-1)\xi}{1+\eta\lambda}. \tag{75}$$

If the sender makes this offer for any outside option value v^o with $\frac{\eta(\lambda-1)}{1+\eta}\xi < v^o \leq \eta\lambda\xi$, the receiver can infer v^o from it. Planning acceptance in period 1 is then optimal for her if

$$t^s \geq v^o - \eta(\lambda - 1)\xi. \tag{76}$$

The restriction $v^o \leq \eta\lambda\xi$ ensures that this inequality is satisfied, which completes the proof. **Step 3.** We show that if $v^o \leq \frac{\eta(\lambda-1)}{1+\eta}\xi$, then the receiver would accept the early offer $(v^s, t^s) = (0, 0)$, provided that the offers for any $v^o > \frac{\eta(\lambda-1)}{1+\eta}\xi$ are those indicated in Step 1 and Step 2. Note that if $v^o \leq \frac{\eta(\lambda-1)}{1+\eta}\xi$, then planning acceptance of $(0, 0)$ is optimal for the receiver. It is then also optimal for her to accept $(0, 0)$ in period 2 if

$$(1+\eta)v^o - \eta(\lambda - 1)\xi \leq 0, \tag{77}$$

which is guaranteed by the upper bound on v^o. This completes the proof of the desired statement. **Step 4.** The results in Steps 1 to 3 characterize the equilibrium outcome that we stated in Proposition 5. In order to prove that any sender-preferred equilibrium exhibits this outcome, it remains to show that the sender's equilibrium payoff cannot be further increased by making alternative offers at outside option values $v^o \in (\eta\lambda\xi, 1]$. For this, we consider two cases. First, we show that, for any $i \in \mathbb{N}$, it does not pay off for the sender to replace the offer (v_i^s, t_i^s) by offers that signal the precise outside option value to the receiver for all values $v_o \in V_i$. The minimal total value of such an offer at outside option value v^o in equilibrium would have to be

$$v^o - \eta(\lambda - 1)\xi, \tag{78}$$

and the corresponding average total value offered for outside options in the set V_i would be

$$\frac{1}{2}(\bar{v}_i + \underline{v}_i) - \eta(\lambda - 1)\xi \tag{79}$$

The fact that $v^o > \eta\lambda\xi$ ensures that this value is weakly larger than w_i^s as defined in equation (71), which proves the claim. Second, assume by contradiction that there is a sender-preferred equilibrium σ in which there is an open set $V \subset (\eta\lambda\xi, 1]$ of outside option values for which the sender's offer signals the precise outside option value. Let V_i be the set of outside option values below those of V so that the sender makes the offer (v_i^s, t_i^s) for all $v^o \in V_i$. By the first case, this set must exist and (since σ is a sender-preferred equilibrium) we must have that w_i^s is defined by equation (71). We show that we can find an equilibrium that yields an even higher expected payoff for the sender. To show this, we change the original equilibrium σ by slightly increasing $\bar{v}_i$ (all else remains the same). Denote by w^s the infimum of the total values offered to outside option values $v^o \in V$. We must have $w^s \geq \bar{v}_i - \eta(\lambda - 1)\xi$ so that we get

$$w^s - w_i^s \geq \frac{\eta(\lambda - 1)}{1 + \eta\lambda}(\bar{v}_i - \eta\lambda\xi). \tag{80}$$

Increasing $\bar{v}_i$ also increases w_i^s and hence the total value offered for outside option values $v^o \in V_i$. We can calculate

$$\frac{\partial w_i^s}{\partial \bar{v}_i} \times (F(\bar{v}_i) - F(\underline{v}_i)) = \frac{1 + \eta}{1 + \eta\lambda} \frac{1}{\frac{1}{2} + \frac{1}{6}\eta(\lambda - 1)} \frac{\eta(\lambda - 1)}{1 + \eta\lambda}(\bar{v}_i - \eta\lambda\xi). \tag{81}$$

This is the marginal increase in the total value that must be provided to secure the acceptance of the sender's offer by outside option values in V_i. The change in the original equilibrium increases the sender's expected payoff if the right-hand side of equation (80) exceeds the right-hand side of equation (81), which is implied by the assumption on the parameters η, λ. This completes the proof of Proposition 5. □

Proof of Proposition 6. We prove the result by construction. For convenience, we abbreviate

$$\Omega = \frac{\eta(\lambda - 1)}{1 + \frac{1}{2}\eta + \frac{1}{2}\eta\lambda}. \tag{82}$$

It is readily verified that $1 > \frac{1}{2}\Omega$. Suppose the sender adopts the following strategy: If $v^o \in [0, \Omega\xi]$, the sender offers $(v^s, t^s) = (0, 0)$ to the receiver. If $v^o \in (\xi\Omega, 1]$, the sender offers $(v^s, 0)$

with $v^s = v^o - \Omega\xi$. The rest of the proof proceeds by steps. **Step 1.** We show that, given the sender's strategy, accepting the sender's offer is the payoff-maximizing plan for the receiver in period 1. Suppose that $v^o > \xi\Omega$ so that the sender's offer is $(v^s, 0)$ with $v^s = v^o - \Omega\xi$. The receiver's expected payoff from the plan "accept the outside option" then equals

$$v^o - \eta(\lambda - 1)\xi, \tag{83}$$

which is strictly smaller than v^s (the expected payoff from accepting the sender's offer). Further, the receiver's expected payoff from the plan "accept the outside option in state 1 and the sender's offer in state 2" is given by

$$v^o - \frac{1}{2}\Omega\xi - \eta(\lambda - 1)\frac{1}{4}\left(1 + \frac{1}{2}\Omega\right)\xi - \eta(\lambda - 1)\frac{1}{4}\left(1 - \frac{1}{2}\Omega\right)\xi. \tag{84}$$

By symmetry, this is also the expected payoff from the plan "accept the outside option in state 2 and the sender's offer in state 1." The term in (84) can be written as

$$v^o - \frac{1}{2}\Omega\xi - \eta(\lambda - 1)\frac{1}{2}\xi. \tag{85}$$

This term is strictly smaller than v^s since $1 + \frac{1}{2}\eta + \frac{1}{2}\eta\lambda > 1$. It remains to show that the acceptance of the sender's offer is also the payoff-maximizing plan if $v^o \leq \xi\Omega$ so that the sender's offer is $(0, 0)$. Note that in this case the expected payoff from the plans mentioned above is weakly smaller than zero as can be seen from the terms in (83) and (84), respectively,when we replace v^o in these terms by the upper bound $\xi\Omega$. This completes the proof of the statement. **Step 2.** We show that, given the sender's strategy, accepting the sender's offer is consistent with a PPE. Suppose that $v^o > \xi\Omega$ so that the sender's offer is $(v^s, 0)$ with $v^s = v^o - \Omega\xi$. Note that we have

$$\frac{1}{2}v^s > \frac{1}{2}v^o - \xi. \tag{86}$$

Therefore, if the receiver planned to accept the sender's offer in period 1, but accepts the outside option in period 2, her payoff equals

$$v^o + \eta\left(\frac{1}{2}v^o + \xi - \frac{1}{2}v^s\right) - \eta\lambda\left(\frac{1}{2}v^s - \frac{1}{2}v^o + \xi\right). \tag{87}$$

This term equals $v^o - \Omega\xi$. Thus, in period 1, the receiver is indifferent between accepting the sender's offer and the outside option. Similarly, we can show that the receiver (weakly) prefers accepting the sender's offer $(0, 0)$ in period 2 if $v^o \leq \xi\Omega$ and she planned to accept this offer in period 1. **Step 3.** We can now construct the desired equilibrium. If at a given outside option value $v^o \in (\xi\Omega, 1]$ the sender deviates from the proposed strategy and offers $(v^s, 0)$ with

$v^s < v^o - \Omega\xi$, then, by Step 2, the receiver would reject this offer. Thus, such a deviation is not profitable. If $v^o \in [0, \Omega\xi]$, the sender earns the maximal possible payoff. Hence, in this case, there is again no profitable deviation for the sender. Finally, assuming optimistic beliefs for off-equilibrium offers ensures that the sender cannot deviate profitably from the proposed strategy. Together with the results from Step 1 and Step 2 this completes the proof. □