Düsseldorf Institute
for Competition Economics

DISCUSSION PAPER

No 07

Quality and welfare in a mixed duopoly with regulated prices: The case of a public and a private hospital

Annika Herr

September 2010

HEINRICH HEINE
UNIVERSITÄT DÜSSELDORF

IMPRINT

DICE DISCUSSION PAPER

Published by
Heinrich-Heine-Universität Düsseldorf, Department of Economics, Düsseldorf Institute for Competition Economics (DICE), Universitätsstraße 1, 40225 Düsseldorf, Germany

Editor:

Prof. Dr. Hans-Theo Normann
Düsseldorf Institute for Competition Economics (DICE)
Phone: +49(0) 211-81-15009, e-mail: normann@dice.uni-duesseldorf.de

DICE DISCUSSION PAPER

ISSN 2190-9938 (online) – ISBN 978-3-86304-006-2

Quality and welfare in a mixed duopoly with regulated prices: The case of a public and a private hospital*

Annika Herr
Düsseldorf Institute for Competition Economics†and
Universität Erlangen-Nürnberg and
Ruhr Graduate School in Economics

September 2010

Abstract

Hospital markets are often characterised by price regulation and the existence of different ownership types. Using a Hotelling framework, this paper analyses the effect of heterogeneous objectives of the hospitals on quality differentiation, profits, and overall welfare in a price regulated duopoly with exogenous symmetric locations. In contrast to other studies on mixed duopolies, this paper shows that in this framework privatisation of the public hospital may increase overall welfare. This holds if the public hospital is similar to the private hospital or less efficient and competition is low. The main driving force is the single regulated price which induces under-(over-)provision of

*I am grateful to Justus Haucap, Hendrik Schmitz, Robert Nuscheler, Tobias Wenzel, and two anonymous referees for valuable comments on my research. This research had been mainly conducted at the Universität Erlangen-Nürnberg and the Ruhr Graduate School in Economics. Financial support by the Alfried Krupp von Bohlen und Halbach-Stiftung is gratefully acknowledged.

†Düsseldorf Institute for Competition Economics, Universität Düsseldorf, Universitätsstr. 1, 40225 Düsseldorf, Germany, annika.herr@dice.uni-duesseldorf.de

quality of the more (less) efficient hospital compared to the first-best. However, if the public hospital is sufficiently more efficient and competition is fierce, a mixed duopoly outperforms both a private and a public duopoly due to an equilibrium price below (above) the price of the private (public) duopoly. This medium price discourages overprovision of quality of the less efficient hospital and –together with the non-profit objective– encourages an increase in quality of the more efficient public hospital.

KEYWORDS: mixed oligopoly, price regulation, quality, hospital competition.
JEL: L13, I18, H42

1 Introduction

As in other countries, public, non-profit and private (for-profit) hospitals compete with each other in Germany. Furthermore, an increasing number of public hospitals have been privatised over the last decade. Since the health care system is mainly publicly financed, regulatory authorities are interested in cost reducing and quality enhancing activities of the hospitals. This article analyses in a theoretical framework, whether and in which respect different objectives lead to different quality outcomes. Furthermore, given the assumed incentive structure, it shows whether and when a mixed duopoly would be preferred to a symmetric public or private duopoly from a welfare perspective.

A mixed oligopoly is in general defined as a market in which two or more firms with different objectives co-exist.[1] In their seminal paper on mixed oligopolies, Merrill and Schneider (1966) assume that the public firm maximises output facing a budget constraint. Often, the public firm is assumed to follow the public owner's interest and to maximise social surplus (De Fraja and Delbono, 1989; Cremer et al., 1991; Nishimori and Ogawa, 2002; Matsumura and Matsushima, 2004; Willner, 2006; Ishida and Matsushima, 2009, e.g.). One issue inherent to that assumption lies in the multiple principal

[1]For surveys of the literature on mixed oligopolies compare De Fraja and Delbono (1990) and Nett (1993).

agent problems a hospital faces. Furthermore, White (2002) shows that a public firm's welfare maximising objective function may be politically manipulated enabling the government to disguise an unpopular agenda. As Cutler (2000) notes, key considerations in the choice of organisational form for hospitals include underlying concerns about agency problems and asymmetric information, the provision of public goods, and access to capital. At the same time, interests of major stakeholders, including administrators, staff, trustees, and communities may also play a role when choosing the ownership of a hospital.[2]

To analyse the behaviour of firms in mixed oligopolies, mostly Cournot or Bertrand models are applied assuming that goods are homogeneous and prices can be set by the firms according to their objective functions. Although the assumptions about the firms' differences in costs and efficiency, number of firms, locations, and timing may matter, it typically turns out that better allocations are achieved when public firms are present (Cremer et al., 1989), where in some cases the welfare-maximising first-best result can be attained. With endogenous costs for investments into efficiency gains, a public monopoly would be preferred to a mixed duopoly (Nishimori and Ogawa, 2002).

In this work, the goods (the treatments of the patients) are assumed to be differentiated à la Hotelling (Hotelling, 1929).[3] Cremer et al. (1991) apply a price-location game where the public firm pays higher wages and maximises social surplus under a non-negative profit constraint. They show that only for less than three and for more than five firms in the market, a mixed oligopoly with less than $(n + 1)/2$ public firms leads to higher welfare than a private oligopoly. If $n = 2$ the location sub-game even yields first-best locations in a mixed duopoly compared to an inefficient private duopoly. Here, we look at quality competition where the welfare implications, which differ from e.g. Cremer et al. (1991) and others, come from the welfare-maximising price regulation, not from a location sub-game. However, with

[2]In his comprehensive review Sloan (2000) classifies and evaluates the theoretical and empirical literature on non-profit hospitals' behaviour until 2000.

[3]Gabszewicz et al. (2001) provide a comprehensive overview over location choice models.

endogenous production costs, privatisation of the public firm would improve welfare compared to a mixed duopoly because it would mitigate the loss arising from excessive cost-reducing investments of the private firm (Matsumura and Matsushima, 2004).

In price regulated markets such as the hospital industry, firms rather compete in quality or location than in prices (Brekke, 2004; Brekke et al., 2006). They model competition in location and quality between two profit maximising hospitals in a price regulated market. The following analysis builds on this model. It is then applied on a mixed duopoly where we assume that locations are symmetric and exogenous. We assume that hospitals cannot change their location in the short or medium term because of their size and infrastructural needs and local demand. More specific, in a first stage, the regulator sets one welfare-maximising price before two hospitals compete in different market structures. Furthermore, the hospitals may not only differ by ownership type but also by marginal costs of production. In a recent study which is most similar to our model, Sanjo (2009) builds on Montefiori (2005) but differentiates between partially private and private hospitals under uncertainty. He shows that the quality of the partially privatised hospital becomes higher than that of the private hospital when the patient's preference for quality is high. Our study additionally discusses the optimal price regulation scheme and the endogenous choice of a market structure by a welfare-maximising regulator.

As in other studies, here the public hospital is assumed to maximise a linear combination of both its profits and its market share. It is necessary for public hospitals to make profits in an environment with increasing costs and decreasing public resources spent on hospitals like in the German hospital market. Furthermore, a high market share may be important to a public hospital for two reasons: first, a high market share reflects a high patients' utility which may reflect an underlying bureaucratic consideration of the public owner. Second, the bigger the hospital, the more power the managers have. This objective function is considered to be more realistic and to mirror the interests of the different stakeholders better than the assumption of welfare-maximising behaviour of public hospitals.[4] In one of the few studies on objectives of different ownership types, Horwitz and Nichols (2009) find that non-profit hospitals rather maximise their own output than profits while the effects on governmental hospitals are smaller and not significant in the U.S..

The main results of this study can be summarised as follows: The mixed duopoly may be optimal compared to a pure profit-maximising or a pure public duopoly if the public firm is more efficient and competition is intense. In the mixed oligopoly, the common regulated price is higher than in the case of public duopoly and lower than in the case of private duopoly. This single regulated price induces under-(over-)provision of quality of the more (less) efficient hospital compared to the first-best quality outcome. However, if the public hospital is sufficiently more efficient and competition is fierce, a mixed duopoly outperforms both a private and a public duopoly due to an equilibrium price below (above) the price of the private (public) duopoly. This medium price discourages overprovision of quality of the less efficient hospital and –together with the non-profit objective– encourages an increase in quality of the more efficient public hospital. The same holds for a private duopoly if the sufficiently more efficient private hospital changes into public ownership: the price will decrease and welfare in the mixed duopoly may be highest.

In contrast to other studies on mixed duopolies, this paper also shows that privatisation of the public hospital may increase overall welfare. This holds if the public hospital is similar to the private hospital or less efficient and competition is low. First-best can be reached in the private duopoly and the public duopoly with minimal quality differentiation if the hospitals are homogeneous. In the mixed duopoly, the regulated price may only induce first best if the public hospital is sufficiently more efficient than the private

[4]We could also imagine that a public hospital tries to maximise its patients' surplus with respect to a budget constraint. Then, the public hospital would always choose the quality level such that the budget constraint is binding. This may lead to overprovision of quality and to an exit of the private hospital. Furthermore, the public hospital would have a quasi-leader role in the mixed duopoly. We do not think that this framework captures the increasing pressure public hospitals face due to an increasing number of private (privatised) hospitals in the German hospital market.

hospital.

Whilst prices and profits are easy to observe, it is difficult to measure a hospital's quality empirically. The measurement of quality in studies of hospital competition has been in the focus of recent research (McClellan and Staiger, 2000b; Romano and Mutter, 2004; Gaynor, 2006). In Germany, quality regulation has been intensified significantly over the last ten years (introduction of minimum quantities, external quality comparisons, and internal quality management as well as the obligation to publish quality reports). However, the evaluation of these means has only started recently and has not led yet to significant results with respect to quality differences between different hospital owners (Geraedts, 2006). Empirical studies of US hospitals find only weak evidence that private hospitals provide higher quality in some local markets (McClellan and Staiger, 2000a). However, their results suggest that other factors than ownership may be more important to explain differences in quality across hospitals. In one of the few studies considering ownership mix, Santerre and Vernon (2006) find that more quality of care per dollar could be generated by increasing for-profit activity in inpatient care and non-profit activity in outpatient care in some market areas in the US. They measure quality indirectly using different utilisation measures under the assumption that utilisation increases when the benefits of quality outweigh its costs at the margin.

This article proceeds as follows. In Section 2, preliminary assumptions will be shortly described. In Section 3, the quality choice of the two hospitals in the three scenarios (private profit-maximising duopoly, state-owned duopoly and mixed duopoly) will be analysed and the comparative statics characteristics of the quality choice in equilibrium will be discussed. Finally, welfare-maximising prices will be derived in Section 4. The corresponding welfare levels, consumer rent, and profits in all three scenarios will be compared with each other and with the first-best scenario in Section 5 before Section 6 concludes.

2 The Structure of the Model

Assume that the two hospitals face a unit mass of patients, distributed uniformly on the line segment $[0, 1]$. Locations $x_i, \ \ i = 1, 2$, are assumed to be exogenously fixed in the hospital sector. Horizontal differentiation may also be understood as specialisation versus diversification of the medical programs the hospitals offer. The only parameter hospitals can choose according to their respective maximisation problems is quality q_i given regulated price p. Marginal production costs c_i differ between the two hospitals and are constant with $p > c_i, i = 1, 2$. Let total marginal costs of production $C = c_1 + c_2$ and the difference in marginal costs or efficiency $D = c_1 - c_2$ where c_1 and c_2 are exogenously given. The framework of Brekke et al. (2006) is generalised by assuming that the two hospitals may differ with respect to their marginal costs c_i (Cremer et al., 1989). Empirically, the difference in marginal costs D can be measured by differences in cost efficiency. In Germany, private hospitals do not underlie the same regulatory restrictions as public or non-profit hospitals. They are, in contrast to public hospitals, not obliged to pay the rather high public sector wages, for example. However, given the input prices and levels of output produced, cost *inefficiency* has been shown to be highest in private hospitals in the US (for a survey compare Hollingsworth (2003, 2008)) and in Germany (Herr, 2008). This means that marginal costs of production (which capture technical and allocative inefficiency) are on average lower in public hospitals than in private hospitals given input use, input prices, and output. Interestingly, private hospitals have been shown to be more profit efficient than public hospitals, though (Herr et al., 2010). Transportation costs, which the patients face, are quadratic in the distance between the patient's location z and the hospital i, i.e. $t(z - x_i)^2$.[5]

[5]Linear transportation cost would lead to similar results.

Utility function and the indifferent patient

A patient located at z derives the utility from getting one unit of the service provided by hospital i located at x_i and providing the quality q_i

$$U(z, x_i, q_i) = v + q_i - t(z - x_i)^2 - p, \tag{1}$$

with price $p > 0$ and transportation costs $t > 0$. In this model, the patient pays the price per treatment either privately or for example as a co-payment to the health insurance. Clearly, the higher the quality the higher is a patient's utility. Furthermore, the constant valuation of consuming the good v is assumed to be sufficiently high such that the market is covered at any time. Due to the latter, a monopolistic hospital would always choose zero quality as long as it is costly (unless otherwise regulated). A monopolist would earn non-negative profits as long as the regulated price exceeds marginal costs of production.[6] We concentrate our analysis on equilibria in pure strategies[7] and assume throughout that $x_2 > x_1$, namely $x_1 \in [0, \frac{1}{2} - \bar{x}]$, $x_2 \in [\frac{1}{2} + \bar{x}, 1]$, with $\bar{x} > 0$ and the two hospitals are located symmetrically, i.e. $x_2 = 1 - x_1$. Then, $x_1 = \frac{1}{2}(1 - \Delta)$ and $x_2 = \frac{1}{2}(1 + \Delta)$ with distance $\Delta = x_2 - x_1$. That means that the indifferent patient is located at

$$\bar{z} = \frac{1}{2} + \frac{q_1 - q_2}{2t\Delta} \tag{2}$$

Profit Functions

As in Brekke et al. (2006), the marginal production costs of one good and the costs of producing a certain quality can be linearly separated, where quality

[6] Brekke et al. (2009) compare a monopolistic altruistic hospital with a market composed by two altruistic hospitals assuming that a fraction of patients may not be treated due e.g. high transportation costs and capacity constraints. They find that it depends on the hospital's valuation of consumer surplus as to which setting would be preferred by the regulator.

[7] Bester et al. (1996) show that the Hotelling location game with quadratic transportation costs and price competition possesses an infinity of mixed strategy Nash equilibria. In these equilibria coordination failure invalidates the principle of "maximum differentiation" discovered by d'Aspremont et al. (1977). For a similar finding, compare Wang and Yang (2001) showing the existence of mixed equilibria in a 2 stage price-quality game.

costs are the costs of investing into higher quality that are not related to the marginal cost of production.[8] The cost of investing into higher quality is assumed to be quadratic throughout the analysis to ensure that the profit function is concave and a unique maximum exists. The profit of hospital i is defined as

$$\pi_i = (p - c_i)y_i - \frac{1}{2}q_i^2 \tag{3}$$

The hospitals' market shares are defined as $y_1 = \bar{z}$ and $y_2 = 1 - \bar{z}$.

The structure of the game is as follows: In stage 0, symmetric locations and marginal costs of production are exogenously fixed before prices are set by the regulatory authority in stage 1 and hospitals compete in quality in stage 2. The game will be solved by backward induction to identify a subgame perfect Nash-equilibrium.

The Three Scenarios

In general, a hospital's objective function is defined as $Z_i = \pi_i + \alpha_i y_i$.[9] Here, as opposed to private profit-maximising hospitals, public hospitals are assumed to maximise their own profits plus a fraction of their market share. In the three possible scenarios the two hospitals behave as follows.

1. Scenario *PD* (profit-maximising duopoly): $\alpha_1 = \alpha_2 = 0$

   As in Brekke et al. (2006), both hospitals behave as profit-maximising private hospitals and maximise their respective objective function $Z_i^p = \pi_i$, $i = 1, 2$.

2. Scenario *SD* (state-run duopoly): $\alpha_1 = \alpha_2 = \alpha > 0$

[8] Bardey et al. (2010) assume the hospital's quality to also determine variable costs. It is not feasible to derive meaningful analytic solutions in this framework with heterogeneous firms. However, if we assume quality to only determine variable costs (not fixed costs) as in Sanjo (2009), the threshold that the public hospital will provide higher quality than the private hospital is $\frac{p}{c_2} > 1$ higher than in the present model derived below.

[9] If we rescale the objective function and assume that the public firm weights the profit motive less than the private firm, and thus $Z_1 = \beta\pi_1 + \bar{z}$ with $\beta \in \{0, 1\}$ the quality and welfare levels do not change in the two symmetric settings since they are equal independent of α (or β) in equilibrium. However, in the mixed duopoly, the public firm will provide even higher quality than in our model. In general, all results go through except that we need to replace α with $1/\beta$ in the mixed duopoly.

Both public hospitals follow the objective function $Z_i^s = \pi_i + \alpha y_i$.

3. Scenario *MD* (mixed duopoly): $\alpha_1 = \alpha > \alpha_2 = 0$

   In this scenario, the mixed duopoly is analysed. It is assumed that hospital 2 is a profit-maximising private hospital, $Z_2^a = \pi_2$, while hospital 1 is a public hospital maximising the mixed objective function $Z_1^a = \pi_1 + \alpha y_1 = \pi_1 + \alpha \bar{z}$.

3 Quality Choice in the Three Scenarios

In all three scenarios, the two hospitals choose their quality levels in equilibrium such that the first order conditions $\frac{dZ_i}{dq_i} = 0$ are fulfilled. Thus, the hospital's quality level in the Nash-equilibrium can be derived to be

$$q_i = \frac{p - c_i + \alpha_i}{2t\Delta}, \tag{4}$$

which is uniquely defined since $\frac{d^2 Z_i}{dq_i^2} < 0$ and $t > 0$, $\Delta > 0$, $p > c_i$. The hospital i's quality level in equilibrium does not depend on the other hospital's quality. It only depends on the price mark-up, the patients' transportation costs and distance and the weight α_i. This equilibrium quality level is a dominant strategy for both hospitals. The first hospital provides higher quality ($q_1 > q_2$) if $D < \alpha_1 - \alpha_2$, i.e. if the cost difference is smaller than the difference in the weights.

If $\alpha_i = 0$, $i = 1, 2$, the equilibrium collapses to a private profit-maximising duopoly (*Scenario PD*) in which the first hospital sets higher quality as long as $D < 0$ and vice versa. In *Scenario SD* the two public hospitals will produce higher quality than in Scenario PD, since they value market shares and thus patient's utility more than purely profit maximising hospitals.

The additional asymmetry of the mixed duopoly (*Scenario MD*) comes from the assumption that $\alpha_1 = \alpha > 0$ for the first hospital and $\alpha_2 = 0$ for the second (pure profit maximiser). Then, $q_1^a > q_2^a$ if $\alpha > D$. Put differently, depending on the underlying cost structure and on α it is possible that the private hospital produces at a higher quality level than the public hospital.

The comparative statics in the three scenarios only differ in their magnitudes of α_i and the levels of qualities and price. The higher the distance Δ or the higher the transportation costs, the lower the two quality levels in equilibrium. These results comply with basic competition theory. When hospitals are close to each other (geographically or in the services they offer) and switching is cheap, competition becomes fierce and quality increases, especially if α_i, the valuation of the market share, is high (see (2)).

As expected, an increase in the price cost margin will lead to higher quality levels for both hospitals. This result holds independent of α_i. Finally, an increase in the weight of the market share α leads to an increase in quality provided.

4 Regulating Prices

In the first step of the game the regulatory authority sets welfare-maximising prices in each of the three scenarios.[10] The corresponding second-best results are compared to the first-best that will be derived first. In general, total welfare is defined as $W = K + \pi_1 + \pi_2$,[11] with

$$K = v - \frac{1}{12}t - p + \frac{1}{2}(q_1 + q_2) + \frac{1}{4}t\Delta(1 - \Delta) + \frac{1}{4\Delta t}(q_2 - q_1)^2 \tag{5}$$

Note that overall welfare does not depend on the price chosen by the regulatory authority. Only the distribution of rents between consumers and producers differs with the price. The computations of equilibrium levels of quality, price, market shares, profits, consumer rent, and welfare discussed in the following can be found in Table 1 in the Appendix.

[10] Conrad (2008) presents a game, where the regulator pays subsidies to give optimal incentives to the private firm to invest into more energy-efficient engines.

[11] Regulated prices as well as resulting quality, profits and consumer rent in equilibrium would not differ when adding the higher utility of the public hospital(s) to overall welfare ($W + \sum_i \alpha_i \bar{y}_i, i = 1, 2$). However, total welfare would be higher than before when there are one or two public hospitals in the market.

4.1 First-best Solution

The welfare maximising first-best quality levels are given by,

$$q_i^w = \frac{t\Delta - 1 - D}{2(t\Delta - 1)} \quad (6)$$

$$q_2^w = \frac{t\Delta - 1 + D}{2(t\Delta - 1)}, \quad (7)$$

where $D = c_1 - c_2$, $t\Delta > \frac{1}{2}$ for a local maximum to exist and $t\Delta > 1 + |D|$ or $t\Delta < 1 - |D|$ for both quality levels to be non-negative. The latter two restrictions ensure that a finite quality level exists ($t\Delta \neq 1$). However, the determinant of the Hessian matrix of the welfare function is only positive if $t\Delta > 1$ excluding the last restriction.

In general, an equilibrium is subgame perfect if and only if the resulting profits, market shares, and quality levels are non-negative which we can assure by assuming that distance and transportation costs are sufficiently high. This ensures concavity of the objective functions.

The difference between the quality levels is $q_1^w - q_2^w = -\frac{D}{t\Delta - 1}$, which only depends on transportation costs, the hospitals' locations and their marginal costs. In the optimum, the public hospital's quality is higher than the private hospital's if $c_1 < c_2$ since $t\Delta > 1$.

If marginal costs are equal for both hospitals, the welfare maximising quality levels are $q_1 = q_2 = \frac{1}{2}$ for both hospitals. Market shares are non-negative if $t\Delta > 1 + |D|$ (non-negative quality) and $t\Delta(t\Delta - 1) > D > t\Delta(1 - t\Delta)$. We assume that in equilibrium, hospitals should at least be able to produce at a non-negative profit level. Both profits are non-negative if the price mark-up is sufficiently high. That means that if the price is low (for example $p^s = t\Delta + \frac{1}{2}C - \alpha$ of Scenario SD derived below), this is only fulfilled if the restrictive non-zero-profit condition $t\Delta > t\widetilde{\Delta} = 1 + (c_1 + c_2) + \alpha > 1$ holds.[12]

Assumption 1 *In the following, comparisons across all four scenarios including the first-best are drawn under the assumption that* $t\Delta > t\widetilde{\Delta} =$

[12]However, except of in the first-best equilibrium lower thresholds would be sufficient (compare Table 1, Row 10).

$1 + (c_1 + c_2) + \alpha > 1$.

Inserting first best q_1^w and q_2^w results into the maximum welfare level given in Table 1, column 5. Given that $t\Delta > 1$, it can be easily shown that overall welfare in the first-best setting increases, the lower transportation costs and marginal costs and the higher the cost-difference. The latter can be explained by a switch of patients to the more efficient hospital. If the cost difference increases, the quality of the less efficient hospital will decrease and it will thus attract fewer patients given exogenous locations. With respect to increasing distance, welfare increases as long as $\Delta \geq 1/2$ and decreases for a smaller distance.

4.2 Price Regulation

In a second-best setting, hospitals behave according to their objective functions and choose the quality levels derived in Section 3 as opposed to welfare-maximising quality of the first-best setting. This behaviour will be anticipated by the regulator in the first stage when setting the market price. Note that we restrict the regulatory authority to impose a single price for both hospitals. We would reach first best always if the government was able to perfectly discriminate between the hospitals and for example to account for the differences in efficiency. However, in the hospital market we actually see that the hospitals receive the same price for the same treatment adjusted for case-mix severity (payments based on Diagnosis Related Groups).

4.2.1 Prices, Quality, Profits, and Welfare in the Private and the Public Duopoly

In the first stage, the welfare function will be maximised by the price setting authority with respect to the quality choice of the hospitals of the second stage. In the private profit-maximising duopoly (PD) the equilibrium price is higher than in the public duopoly ($p^p = t\Delta + \frac{1}{2}C = p^s + \alpha$) to induce the hospitals to produce at a higher quality level.[13] The resulting quality

[13]The second order conditions are fulfilled in all three scenarios.

levels correspond with each other in the two scenarios with $q_1^p = q_1^s = \frac{1}{2} - \frac{D}{4t\Delta}$ and $q_2^p = q_2^s = \frac{1}{2} + \frac{D}{4t\Delta}$. Thus, the higher price induces both profit-maximising hospitals to produce at the same quality level as if they were also considering their respective market shares in their objective function. The first hospital's quality is lower than the quality of the second hospital if $c_1 > c_2$. A unique Nash equilibrium with non-negative quantities and profits exists if simultaneously $\pi_i \geq 0$ and $y_i \geq 0$ at equilibrium prices and quality levels.

Proposition 1 *Let $p^s = t\Delta + \frac{1}{2}C - \alpha$ and $t\Delta > \alpha + \frac{1}{2}C + \frac{1}{2}$ with two public hospitals. Then, a unique Nash equilibrium with non-negative quantities and profits exists. In the private duopoly with $p^p = t\Delta + \frac{1}{2}C$, it suffices that $t\Delta > \frac{1}{2}|D| + \frac{1}{4}$ for a subgame-perfect Nash-equilibrium in pure strategies to exist.*

Since the second best quality levels are equal across scenarios, welfare is equally high in both symmetric settings ($W^p = W^s$). The distribution of consumer rent and profits differs, though, since the price and profits are lower and the consumer rent is higher if both hospitals are state-run (SD).

4.2.2 Prices, Quality, Profits, and Welfare in the Mixed Duopoly (Scenario MD)

In the mixed duopoly, quality levels differ between the two hospitals. The welfare maximising price lies between the equilibrium prices in the two symmetric duopolies ($p^s < p^a < p^p$). The price will always be higher in the mixed duopoly than in the symmetric public duopoly to induce the private hospital to produce at a higher quality level. Additionally, this price increase reduces the underprovision of quality of the more efficient public firm. For positive market shares of both hospitals, $2(t\Delta)^2 > D - \alpha$ and $2(t\Delta)^2 > -(D - \alpha)$ need to be assured which is given since $p > c_i \Leftrightarrow 2t\Delta > \alpha + |D|$ and $t\Delta > 1$. The corresponding quality levels $q_1^a = \frac{1}{2} - \frac{D-\alpha}{4t\Delta}$ and $q_2^a = \frac{1}{2} + \frac{D-\alpha}{4t\Delta}$ are higher and lower, respectively, than the levels in the two symmetric scenarios.

Proposition 2 *Let $p^a = t\Delta + \frac{1}{2}C - \frac{1}{2}\alpha$ and $t\Delta > \alpha + \frac{1}{2}C + \frac{1}{2}$. Then, a unique Nash equilibrium with non-negative quantities and profits exists in the mixed* *duopoly. The public hospital's quality is higher than the private hospital's if $D < \alpha$, i.e. if the difference in marginal costs is lower than the valuation of the market share. The private hospital earns higher profits than the public hospital if $D < \alpha \frac{\alpha - t\Delta}{\alpha - t\Delta - 2t^2\Delta^2}$ which is possible even if $D > \alpha$.*

The eventual welfare level in the asymmetric mixed duopoly W^a is given in Table 1, Row 9.

5 Comparison of Welfare, Consumer Surplus, and Profits

Assume in the following that $t\Delta > \widetilde{t\Delta} = 1 + C + \alpha$ to enable comparisons across all four scenarios (including the first-best scenario). Furthermore, let $c_i^p = c_i^s = c_i^a = c_i$, $i = 1, 2$. This assumption applies also when a hospital changes the ownership. That means that marginal costs of production do not alter after a switch from, for example, public to private ownership. In the following, all results are interpreted given this hypothetical set-up.

5.1 Comparison of Welfare Levels

Given second best prices in the two symmetric scenarios, quality and welfare levels are of the same magnitude, no matter whether hospitals take into account market shares or only maximise profits. Furthermore, it can be shown that

$$W^a > W^p = W^s \Leftrightarrow D < -\alpha \frac{t\Delta - 1}{2(1 + t\Delta)}$$

with $D = c_1 - c_2$ and $t\Delta > \widetilde{t\Delta} > 1$. Let $D > 0$. Then, the welfare level in the mixed duopoly is below the level in the two symmetric scenarios. In this case, a private duopoly would provide higher welfare than a mixed market due to its symmetric structure. Conversely, there is a difference in marginal costs D, for which a mixed duopoly increases welfare compared to two public or two private hospitals. The lower the valuation of the market share α (since $t\Delta > 1$) or the more intense the competition (low $t\Delta$), a regulatory authority would rather implement a mixed duopoly than a symmetric setup as long as the public hospital has an advantage in marginal cost of production. That means, in a symmetric private duopoly the hospital with lower costs of production (higher efficiency) should switch the ownership type to reduce under-provision of quality. The resulting lower price additionally gives incentives to reduce over-provision of quality by the less efficient hospital.

Naturally, the first-best setting gives the highest welfare level since with $t\Delta > \widetilde{t\Delta} > 1$ the comparison shows

$$\begin{aligned} W^s - W^w &= W^p - W^w = -\frac{D^2\left(t\Delta+1\right)^2}{16(t\Delta)^3(t\Delta-1)} \\ W^a - W^w &= -\frac{\left(\alpha(1-t\Delta) - D\left(t\Delta+1\right)\right)^2}{16(t\Delta)^3\left(t\Delta-1\right)} \end{aligned}$$

The first-best result can be reached in the symmetric Scenarios PD and SD if $D = 0$ that means if marginal costs are equal across hospitals. Comparing the two symmetric settings, it is rather a political decision whether the public authority prefers to support producers by privatising both hospitals or to enlarge consumer rent. In the mixed duopoly, the first-best can only be reached if $t\Delta = \frac{\alpha-D}{\alpha+D} > \widetilde{t\Delta}$, thus if $c_1 \ll c_2$. In the case that the public hospital has a big cost advantage, a mixed setting would increase welfare compared to the symmetric settings. The reason lies again in the incentives inherent to the price regime. However, if the private hospital has the efficiency advantage, the first-best outcome cannot be reached in the mixed duopoly.

5.2 Comparison of Consumer Surplus

Since $\frac{\partial K}{\partial q_i} = \frac{q_i-q_j}{2t\Delta} + \frac{1}{2} > 0$ if $q_i - q_j > -t\Delta$, for at least one hospital $i \neq j$ the consumer surplus would be maximal if quality increased to infinity or distance is close to zero (leading to infinitely high quality via high competition between the hospitals).[14] However, given the quality choice by the hospitals and

[14] The consumer surplus and the profits of the three scenarios are not compared with the first-best setting since in the latter any arbitrary price would lead to maximal welfare.

inserting second best prices which are fully paid by the patients, consumer surplus can be derived as shown in Table 1, Row 8.[15]

In Scenario SD ($\alpha > 0$ for both hospitals) the consumer surplus is higher than in the private duopoly, namely

$$K^s = K^p + \alpha, \tag{8}$$

due to higher quality and a lower regulated price. In the mixed duopoly it holds that

$$K^a = K^p + \frac{1}{2}\alpha + \frac{1}{16(t\Delta)^3}\left(\alpha - 2D\right)\alpha \tag{9}$$

Proposition 3 *Assume that a sub-game perfect Nash equilibrium exists where both hospitals are active in the market in all three scenarios, i.e. transportation costs and distance are sufficiently high with* $t\Delta > \frac{1}{2}C + \alpha + \frac{1}{2}$*. Then,* $K^s > K^a > K^p$*.*

For an analysis of consumer rents with lower transportation costs, compare Appendix B.

5.3 Comparison of Profits

The profits of the first two scenarios are easy to compare with each other. Since welfare levels coincide but prices are higher in the duopoly with two profit-maximising hospitals than in the public duopoly, profits will be higher in the former duopoly than in the latter. From Table 1, Rows 6 and 7, it can be shown that $\pi_i^p > \pi_i^s$ if $|D| < 2(t\Delta)^2$. Compared to the mixed duopoly the following Proposition can be derived.

Proposition 4 *Assume that a sub-game perfect Nash equilibrium exists where both hospitals are active in the market, i.e. transportation costs and distance are sufficiently high with* $t\Delta > \frac{1}{2}C + \frac{1}{2} + \alpha$*. Then,* $\pi_i^p > \pi_i^a > \pi_i^s$ *for* $i = 1, 2$*.*

Proof: See Appendix C for a comparison of the respective profit functions.

[15]It may be possible in reality that the contributions to the health insurance do not cover the full price. Then, consumer surplus would even be higher.

6 Conclusion

This analysis has shown that a mixed oligopoly can lead to the highest welfare and quality when compared to two public or two private hospitals and may come closest to the first-best solution. This result implies that it can be best to privatise one or several less efficient public hospitals when a more efficient public hospital is still present in the market. The reason lies in the regulated price which is the same across the hospitals.

Compared to the mixed duopoly, a private duopoly will be preferred if the public hospital that would be privatised faces similar or higher marginal costs than the private competitor, competition is not intense and the public owner would value its market share sufficiently much. Then, an inefficient public hospital would provide extensive quality given a relatively high price which reduces overall welfare.

Our result derived in a price regulated setting conflicts with the result by Cremer et al. (1991) who state that a mixed duopoly would be superior to a private duopoly in a price-location game even if the public firm faces higher wages and thus higher marginal costs. Their result is mainly due to the equilibrium choice of the first-best locations in a mixed duopoly compared to a maximally differentiated private duopoly. Here, without considering the location game, in the mixed duopoly, first-best can only be reached if the public hospital has a big cost advantage compared to the private (for-profit) hospital.

In an extended framework, a hospital could be also viewed as a platform bringing together doctors and patients to internalise the common network externality in this so-called two-sided market (Pezzino and Pignataro, 2008; Bardey et al., 2009).

Further possible generalisations of this model include the introduction of endogenous costs, location choice, choice of slack, and the extension to more than two competitors. For future research on hospital privatisation, it is essential to identify the objectives of different ownership types empirically. Since the empirical literature on hospital competition and ownership-mix is scarce, more studies should be conducted which exploit variation in ownership-mix, e.g. across countries or regions, and in which it is not only accounted for prices and costs, but also for quality.

References

Bardey, D., Canta, C. and Lozachmeur, J.-M. (2010). Health care providers payments regulation when horizontal and vertical differentiation matter, *Working Paper 532*, IDEI.

Bardey, D., Cremer, H. and Lozachmeur, J.-M. (2009). Competition in two-sided markets with common network externalities, *Working Paper 75*, Universidad del Rosario.

Bester, H., De Palma, A., Leininger, W. and Thomas, J. (1996). A noncooperative analysis of Hotelling's location game, *Games and Economic Behavior* **12**: 165–186.

Brekke, K. (2004). Competition or coordination in hospital markets with unionised labour, *International Journal of Health Care Finance and Economics* **4**: 65–89.

Brekke, K., Nuscheler, R. and Straume, O. R. (2006). Quality and location choices under price regulation, *Journal of Economics & Management Strategy* **15**(1): 207–227.

Brekke, K., Siciliani, L. and Straume, O. R. (2009). Hospital competition and quality with regulated prices, *Working Paper 2635*, CESifo.

Conrad, K. (2008). Price competition and product differentiation when goods have network effects, *German Economic Review* **7**(3): 339-361.

Cremer, H., Marchand, M. and Thisse, J.-F. (1989). The public firm as an instrument for regulating an oligopolistic market, *Oxford Economic Papers* **41**(2): 283–301.

Cremer, H., Marchand, M. and Thisse, J.-F. (1991). Mixed oligopoly with differentiated products, *International Journal of Industrial Organization* **9**(1): 45–53.

Cutler, D. M. (2000). Introduction, *in* D. M. Cutler (ed.), *The changing hospital industry: comparing not-for-profit and for-profit institutions*, A NBER conference report, Chicago, chapter 3, pp. 93–112.

d'Aspremont, C., Gabszewicz, J. J. and Thisse, J.-F. (1977). On Hotelling's stability in competition, *Econometrica* **47**(5): 1145–50.

De Fraja, G. and Delbono, F. (1989). Alternative strategies of a public enterprise in oligopoly, *Oxford Economic Papers* **41**: 302–311.

De Fraja, G. and Delbono, F. (1990). Game theoretic models of mixed oligopoly, *Journal of Economic Surveys* **4**(1): 1–17.

Gabszewicz, J. J., Thisse, J.-F., Fujita, M. and Schweizer, U. (2001). *Location Theory*, Routledge, New York.

Gaynor, M. (2006). What do we know about competition and quality in health care markets?, *Working Paper 12301*, NBER.

Geraedts, M. (2006). Auswirkungen von Qualitätsregulierungen auf das Angebot von Krankenhausleistungen, *in* K. J, R. B-P and S. H (eds), *Krankenhausreport 2006- Schwerpunkt: Krankenhausmarkt im Umbruch*, Schattauer, Stuttgart, pp. 187–203.

Herr, A. (2008). Cost and technical efficiency of German hospitals: Does ownership matter?, *Health Economics* **17**: 1057–1071.

Herr, A., Schmitz, H. and Augurzky, B. (2010). Profit efficiency and ownership of German hospitals, *Health Economics*, **published online: June 7, 2010**.

Horwitz, J. R. and Nichols, A. (2009). Hospital ownership and medical services: Market mix, spillover effects, and nonprofit objectives, *Journal of Health Economics* **28**: 924–937.

Hotelling, H. (1929). Stability in competition, *Economic Journal* **39**: 41–57.

Ishida, J. and Matsushima, N. (2009). Should civil servants be restricted in wage bargaining? A mixed-duopoly approach, *Journal of Public Economics* **93**: 634–646.

Matsumura, T. and Matsushima, N. (2004). Endogenous cost differentials between public and private enterprises: A mixed duopoly approach, *Economica* **71**(284): 671–688.

McClellan, M. B. and Staiger, D. O. (2000a). Comparing hospital quality at for-profit and not- for-profit hospitals, *The Changing Hospital Industry: Comparing For-Profit and Not-for-Profit Institutions*, NBER Chapters, 93–112.

McClellan, M. and Staiger, D. (2000b). Comparing the quality of health care providers, *Forum for Health Economics & Policy* **3**: Art. 6.

Merrill, W. and Schneider, N. (1966). Government firms in oligopoly industries: A short-run analysis, *Quarterly Journal of Economics* **80**: 400–412.

Montefiori, M. (2005). Spatial competition for quality in the market for hospital care, *European Journal of Health Economics* **6**: 131–135.

Nett, L. (1993). Mixed oligopoly with homogeneous goods, *Annals of Public and Cooperative Economics* **64**(3): 317–322.

Nishimori, A. and Ogawa, H. (2002). Public monopoly, mixed oligopoly and productive efficiency, *Australian Economic Papers* **41**(2): 185–190.

Pezzino, M. and Pignataro, G. (2008). Competition in the health care market: a "two-sided" approach, *Working paper*.

Romano, P. S. and Mutter, R. (2004). The evolving science of quality measurement for hospitals: Implications for studies of competition and consolidation, *International Journal of Health Care Finance and Economics* **4**: 131–157.

Sanjo, Y. (2009). Quality choice in a health care market: a mixed duopoly approach, *European Journal of Health Economics* **10**: 207–215.

Santerre, R. E. and Vernon, J. A. (2006). The consumer welfare implications of the hospital ownership mix in the US: an exploratory study, *Health Economics* **15**: 1187–1199.

Sloan, F. A. (2000). Not-for-profit ownership and hospital behavior, *in* A. J. Culyer and J. Newhouse (eds), *Handbook of Health Economics 1B*, Elsevier, Amsterdam, chapter 21, pp. 1141–1174.

Wang, X. H. and Yang, B. Z. (2001). Mixed-strategy equilibria in a quality differentiation model, *International Journal of Industrial Organization* **19**(1-2): 213 – 226.

White, M. D. (2002). Political manipulation of a public firm's objective function, *Journal of Economic Behavior & Organization* **49**: 487–499.

Willner, J. (2006). A mixed oligopoly where private firms survive welfare maximisation, *Journal of Industry, Competition and Trade* **6**: 235–251.

A Table with main results

	Profit-max. duopoly (PD)	State-run duopoly (SD)	Mixed Duopoly (MD)	First-best
q_1	$q_1^p = \frac{p-c_1}{2t\Delta} = \frac{1}{2} - \frac{D}{4(t\Delta)}$	$q_1^s = \frac{p-c_1+\alpha}{2t\Delta} = \frac{1}{2} - \frac{D}{4(t\Delta)}$	$q_1^a = \frac{p-c_1+\alpha}{2t\Delta} = \frac{1}{2} - \frac{D-\alpha}{4(t\Delta)}$	$q_1^w = \frac{1}{2} - \frac{D}{2(t\Delta-1)}$
q_2	$q_2^p = \frac{p-c_2}{2t\Delta} = \frac{1}{2} + \frac{D}{4(t\Delta)}$	$q_2^s = \frac{p-c_2+\alpha}{2t\Delta} = \frac{1}{2} + \frac{D}{4(t\Delta)}$	$q_2^a = \frac{p-c_2}{2t\Delta} = \frac{1}{2} + \frac{D-\alpha}{4(t\Delta)}$	$q_2^w = \frac{1}{2} + \frac{D}{2(t\Delta-1)}$
p	$p^p = t\Delta + \frac{1}{2}C$	$p^s = t\Delta + \frac{1}{2}C - \alpha$	$p^a = t\Delta + \frac{1}{2}C - \frac{1}{2}\alpha$	$p^w > c_1, p^w > c_2, p^w \geq p^s$
$\bar{z}$	$\frac{1}{2} - \frac{D}{4(t\Delta)^2}$	$\frac{1}{2} - \frac{D}{4(t\Delta)^2}$	$\frac{1}{2} - \frac{D-\alpha}{4(t\Delta)^2}$	$\frac{1}{2} - \frac{D}{2t\Delta(t\Delta-1)}$
π_1	$\pi_1^p = \frac{1}{32(t\Delta)^2} D\left(-4t\Delta + 3D\right)$ $-\frac{1}{8}\left(-4t\Delta + 2D + 1\right)$	$\pi_1^s = \frac{1}{32(t\Delta)^2} D\left(8\alpha - 4t\Delta + 3D\right)$ $-\frac{1}{8}\left(4\alpha - 4t\Delta + 2D + 1\right)$	$\pi_1^a = \frac{\alpha-D}{32(t\Delta)^2}\left(4t\Delta - 5\alpha - 3D\right)$ $+\frac{1}{8}\left(4t\Delta - 2\alpha - 2D - 1\right)$	$\pi_1^w = (p-c_1)\left(\frac{1}{2} - \frac{D}{2t\Delta(t\Delta-1)}\right)$ $-\frac{1}{8(t\Delta-1)^2}\left(t\Delta - 1 - D\right)^2$
π_2	$\pi_2^p = -\frac{1}{32(t\Delta)^2} D\left(-4t\Delta - 3D\right)$ $-\frac{1}{8}\left(-4t\Delta - 2D + 1\right)$	$\pi_2^s = -\frac{1}{32(t\Delta)^2} D\left(8\alpha - 4t\Delta - 3D\right)$ $-\frac{1}{8}\left(4\alpha - 4t\Delta - 2D + 1\right)$	$\pi_2^a = -\frac{\alpha-D}{32(t\Delta)^2}\left(4t\Delta - 3\alpha + 3D\right)$ $+\frac{1}{4}\left(2t\Delta - \alpha + D - \frac{1}{2}\right)$	$\pi_2^w = (p-c_2)\left(\frac{1}{2} + \frac{D}{2t\Delta(t\Delta-1)}\right)$ $-\frac{1}{8(t\Delta-1)^2}\left(t\Delta - 1 + D\right)^2$
K	$K^p = v - \frac{1}{12}t - \frac{1}{4}t\Delta(3+\Delta)$ $+\frac{1}{2} - \frac{1}{2}C + \frac{1}{16t^3\Delta^3}D^2$	$K^s = K^p + \alpha$	$K^a = K^p + \frac{1}{2}\alpha + \frac{(\alpha-2D)\alpha}{16(t\Delta)^3}$	–
W	$W^p = v - \frac{1}{12}t + \frac{1}{4}t\Delta(1-\Delta)$ $+\frac{1}{4} - \frac{1}{2}C + \frac{1}{16\Delta^3 t^3}\left(3t\Delta + 1\right)D^2$	$W^s = W^p$	$W^a = v - \frac{1}{12}t - \frac{1}{2}C + \frac{1}{4} + \frac{(\alpha-D)^2}{16t^3\Delta^3}$ $+\frac{1}{4}t\Delta\left(1-\Delta\right) - \frac{t\Delta(\alpha-D)(\alpha+3D)}{16t^3\Delta^3}$	$W^w = v - \frac{1}{12}t - \frac{1}{2}C + \frac{1}{4}$ $+\frac{1}{4}t\Delta(1-\Delta) + \frac{D^2}{4t\Delta(t\Delta-1)}$
n.c.	$t\Delta > \frac{1}{2}\lvert D\rvert + \frac{1}{4}$	$t\Delta > \frac{1}{2} + \frac{1}{2}C + \alpha$	$t\Delta > \frac{1}{2} + \frac{1}{2}C + \alpha$	$t\Delta > 1 + C + \alpha$

Table A-1: n.c.: necessary condition for subgame-perfect equilibrium, difference in marginal costs $D = c_1 - c_2$, total marginal costs $C = c_1 + c_2$, distance $\Delta = x_2 - x_1$.

B Consumer Surplus with high and low Transportation Costs

Comparing the consumer rents without obeying the necessary constraint on transportation costs and distance, we can identify three different orders of magnitude shown in the table below. In the case of high transportation costs (1 and 2), the order is clear, the symmetric public scenario is preferred by the patients with $K^s > K^a > K^p$. For low transportation costs, the asymmetric setting can lead to lowest (3) and highest (4) consumer surplus depending on the relative marginal costs of the two hospitals.

	$D > \frac{1}{2}\alpha$	$D < \frac{1}{2}\alpha$
$t\Delta > \frac{1}{2}\sqrt[3]{(-2D+\alpha)}$ if $D < \frac{1}{2}\alpha$	1	2
$t\Delta > \frac{1}{2}\sqrt[3]{(2D-\alpha)}$ if $D > \frac{1}{2}\alpha$	$K^s > K^a > K^p$	$\mathbf{K}_s > \mathbf{K}_a > \mathbf{K}_p$
$t\Delta < \frac{1}{2}\sqrt[3]{(-2D+\alpha)}$ if $D < \frac{1}{2}\alpha$	3	4
$t\Delta < \frac{1}{2}\sqrt[3]{(2D-\alpha)}$ if $D > \frac{1}{2}\alpha$	$K^s > K^p > K^a$	$K^a > K^s > K^p$

As expected, two profit maximising hospitals set quality levels such that the consumer surplus is always lowest across scenarios. Since it is assumed that $t\Delta > t\tilde{\Delta}$, only cases 1 and 2 will be observed in equilibrium, where Case 2 includes the results for a more efficient public hospital.

C Comparison of Profits

The hospital's profits in the mixed duopoly are lower than the profits of the profit maximising hospitals in the private duopoly if

$$\pi_1^p - \pi_1^a > 0 \Leftrightarrow 8t^2\Delta^2 - 4t\Delta > -5\alpha + 2D$$

and

$$\pi_2^p - \pi_2^a > 0 \Leftrightarrow 8t^2\Delta^2 + 4t\Delta > 3(\alpha - 2D)$$

The profits of the first of the two public hospitals in the state-owned duopoly are lower than the public hospital's profits of the mixed duopoly if

$$\pi_1^s - \pi_1^a < 0 \Leftrightarrow 8t^2\Delta^2 + 4t\Delta > 5\alpha + 6D$$

The profits of the second public hospital are lower than the private hospital's profits of the mixed duopoly if

$$\pi_2^s - \pi_2^a < 0 \Leftrightarrow 8t^2\Delta^2 - 4t\Delta > -3\alpha - 2D$$

In the subgame-perfect Nash equilibrium it is assumed that transportation costs and distance are sufficiently high with $t\Delta > \frac{1}{2}C + \frac{1}{2} + \alpha$. Thus, the above inequalities are fulfilled in equilibrium and $\pi_i^p > \pi_i^a > \pi_i^s$.

PREVIOUS DISCUSSION PAPERS

07 Herr, Annika, Quality and welfare in a mixed duopoly with regulated prices: The case of a public and a private hospital, September 2010.

06 Blanco, Mariana, Engelmann, Dirk and Normann, Hans-Theo, A Within-Subject Analysis of Other-Regarding Preferences, September 2010.

05 Normann, Hans-Theo, Vertical Mergers, Foreclosure and Raising Rivals' Costs – Experimental Evidence, September 2010.

04 Gu, Yiquan and Wenzel, Tobias, Transparency, Price-Dependent Demand and Product Variety, September 2010.

03 Wenzel, Tobias, Deregulation of Shopping Hours: The Impact on Independent Retailers and Chain Stores, September 2010.

02 Stühmeier, Torben and Wenzel, Tobias, Getting Beer During Commercials: Adverse Effects of Ad-Avoidance, September 2010.

01 Inderst, Roman and Wey, Christian, Countervailing Power and Dynamic Efficiency, September 2010.

Heinrich-Heine-University of Düsseldorf

Düsseldorf Institute for Competition Economics (DICE)

Universitätsstraße 1 _ 40225 Düsseldorf
www.dice.uni-duesseldorf.de

ISSN 2190-9938 (online)
ISBN 978-3-86304-006-2