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On the Role of Consumer Expectations in Markets with Network Effects*

Irina Suleymanova† Christian Wey‡

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Abstract

We analyze the role of consumer expectations in a Hotelling model of price competition when products exhibit network effects. Expectations can be strong (stubborn), weak (price-sensitive) or partially stubborn (a mix of weak and strong). As a rule, the price-sensitivity of demand declines when expectations are more stubborn. An increase of stubbornness i) reduces competition, ii) increases (decreases) the parameter region with a unique duopoly equilibrium (multiple equilibria), iii) reduces the conflict between consumer and social preferences for de facto standardization, and iv) reduces the misalignment between consumer and social preferences for compatibility.

JEL-Classification: D43, D84, L13

Keywords: Network Effects, Expectations, Duopoly, Compatibility, Welfare

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1 Introduction

Network effects play an important role in many software and digital markets (Shapiro and Varian, 1999). They arise when consumer utility increases in the number of other consumers using compatible products.[1] Competition between incompatible products leads to strategic uncertainty and coordination problems when consumers' choices are essentially simultaneous. In those settings the formation of expectations about other consumers' purchasing decisions becomes a critical determinant of market performance; or as Farrell and Klemperer (2007, p. 2025) have put it: "As in any game with multiple equilibria, *expectations are key.* If players expect others to adopt, they too will adopt" (emphasis in original). Moreover, the sheer power of consumer expectations may drive market outcomes such that they become self-fulfilling: "In a real sense, the product that is *expected* to become the standard will become the standard" (Shaprio and Varian, 1999, p. 14, emphasis in original).

In this paper we analyze how the formation of consumer expectations affects duopoly competition and market outcomes when firms offer incompatible products. We distinguish between weak and strong expectations.[2] The notion of strong expectations mirrors the fact that consumers may stubbornly favor a particular market outcome. In that case, consumers do not revise their expectations according to firms' pricing decisions. In contrast, we refer to weak expectations when consumers are completely uncommitted to any initial expectations about firms' market shares. Expectations then fully take account of firms' competitive actions.

Our notion of strong expectations builds on Katz and Shapiro (1985) who analyzed a Cournot oligopoly model with network effects. They proposed to solve for the fulfilled expectations Cournot-Nash equilibrium. In their set-up consumers form expectations about firms' network sizes *before* market competition occurs. Hence, when expectations are strong, they do not respond to firms' market actions, so that firms must treat them as given when choosing prices. Yet, rationality requires that expectations are self-fulfilling in equilibrium.

In contrast, weak expectations are fully price-sensitive such that they depend on firms' *actual* competitive behavior. Accordingly, weak expectations are formed *after* firms make their

[1] More generally, network effects may also arise when consumers care about the social influence of their consumption decisions (see Becker 1991, Corneo and Jeanne 1997a/b, or Grilo, Shy, and Thisse 2001).

[2] See Farrell and Klemperer (2007) for a recent survey of the network effects literature.

decisions (see, e.g., Farrell and Saloner, 1992). The weak expectations case is solved under standard Nash equilibrium requirements. As a consequence, weak expectations are fulfilled on- and off-equilibrium.

In contrast to Katz and Shaprio (1985) we analyze the role of expectations under duopolistic price competition with firms offering incompatible products that exhibit positive network effects. Products are horizontally differentiated à la Hotelling. We suppose a linear setting, where network effects are linearly increasing in network size with slope $b > 0$ and horizontal differentiation is linearly increasing in distance with parameter $t > 0$.

We combine both weak and strong expectations within a single model. We analyze both polar cases as well as the "mixed" case. In the latter scenario, consumer expectations are a hybrid of weak and strong expectations. These expectations can be described as partially stubborn. We parameterize the mixed case by λ (with $0 \leq \lambda \leq 1$) which measures the degree by which expectations are weak. Conversely, $1 - \lambda$ indicates the degree of stubbornness.

Our analysis of network effects within a Hotelling model extends the works of Farrell and Saloner (1992), Baake and Boom (2001), and Grilo, Shy, and Thisse (2001).[3] Those papers have exclusively focused on the weak expectations case. We contribute to that literature by analyzing different types of consumer expectations. Our approach allows us to compare different consumer expectations and how they determine market equilibrium.

We find that consumer expectations (varying from weak, $\lambda = 1$, to strong, $\lambda = 0$) together with product differentiation and network effects (both captured by the ratio $\kappa := t/b$) jointly determine the nature of competition and market performance. As a rule, the price-sensitivity of demand declines when consumer expectations become more stubborn. We analyze the following market features: *i)* intensity of competition, *ii)* multiple equilibria, *iii)* preferences for (de facto) standardization,[4] and finally, *iv)* preferences for compatibility.

The following results emerge from our analysis: *i)* The *intensity of competition* (which is inversely proportional to price levels) increases when network effects increase and/or product

[3] See also the textbook exposition in Shy (2001).

[4] The literature distinguishes between de facto and de jure standardization (David and Greenstein 1990). The former is achieved through the market mechanism and the latter is the result of committee agreement or governmental intervention. We focus on de facto standardization which emerges as a monopoly equilibrium outcome in our model.

differentiation decreases. Moreover, competition is more intense when expectations become less stubborn. Intuitively, when expectations are less stubborn, then consumers are more eager to take into account the positive demand effects induced by a price reduction.

ii) Multiple equilibria are an issue when production differentiation is small and/or network effects are large. In those instances, both a duopoly equilibrium and (multiple) monopoly equilibria are possible. Overall, an increase in the degree of stubbornness tends to decrease the parameter range where many equilibria exist, while the parameter range with a unique (duopoly) equilibrium becomes larger. When product differentiation becomes sufficiently large, a unique (duopoly) equilibrium survives for any type of consumer expectations.

iii) We analyze *preferences for standardization* by comparing consumer surplus and social welfare in the monopoly equilibria with those in the duopoly equilibrium. Social welfare is always largest in the monopoly equilibria (if they exist). Consumers tend to favor the duopoly equilibrium over the monopoly equilibria when product differentiation is low, network effects are large, and expectations are weak. Hence, in the latter case consumer and social preferences regarding standardization are not aligned. However, as consumer expectations become more stubborn, price competition is reduced in the duopoly equilibrium which tends to make consumers relatively better off when de facto standardization is realized.

iv) Preferences for compatibility are driven by the differences in consumer surplus and social welfare with incompatible and compatible products. If products are compatible, then a unique duopoly equilibrium exists where both products provide the same amount of network effects. From a social welfare point of view, compatibility is always better than incompatibility because of large network effects and preserved product variety.[5] However, when consumer expectations are sufficiently weak and/or product differentiation is relatively low (or, network effects are relatively high), then consumers prefer products to be incompatible because of lower prices (both in the duopoly and monopoly equilibria). In that case, a consumer surplus standard and a social welfare standard lead to contradicting views concerning the desirability of compatibility. The advantage of lower prices under incompatibility becomes less pronounced as stubbornness increases. In that sense, stubbornness tends to reduce the conflict between both welfare concepts.

Our analysis of network effects within a Hotelling model is most closely related to Grilo,

[5] We abstract from possible costs to implement compatibility.

Shy, and Thisse (2001; henceforth: GST). That paper considers only the weak expectations case. GST's contribution is to analyze strong network effects (termed as "strong conformity"), where they use the invariance axiom to reduce the set of multiple equilibria. We apply this technique to solve the case of weak expectations what yields qualitatively the same results as in GST. In the course of our analysis of the mixed expectations case, we find that the invariance axiom is also applicable here. There are several differences between GST and our analysis: *First*, GST do not model consumer expectations explicitly and, therefore, neither consider strong expectations nor mixed expectations. *Second*, and relatedly, we focus on the comparison of weak, strong, and mixed expectations, and we show how different types of expectations affect market outcomes. *Third*, we analyze the market outcome when products are compatible and we compare that outcome with the equilibria under incompatibility. *Fourth*, in GST both the network effects function and the transportation costs function are quadratic, while we suppose linear specifications.

Our analysis of the role of consumer expectations complements the (largely informal) analysis in Farrell and Katz (1998) who examine how consumer expectations affect firms' compatibility and innovation incentives.[6] They distinguish between expectations which "track surplus" or "stubbornly favor one firm".[7,8] Expectations that track surplus induce consumers to expect the firm to win the market which provides the highest surplus (holding network size constant). Expectations stubbornly favor one firm if consumers expect a particular firm to dominate the market independently of quality and price. Those expectations only make sense when consumers agree which product is preferable. Our analysis is different, because we distinguish between weak and strong expectations in the context of differentiated products, where consumers should have a natural preference for the product which is "closest" in the product-characteristic space. However, our analysis is complementary to Farrell and Katz (1998) as we make the game-theoretic specifications explicit, which induce weak or strong expectations.

[6] R&D incentives in markets with network effects have also been analyzed in Kristiansen and Thum (1997) and Glazer, Kanniainen, and Mustonen (2006) when consumer expecations are weak.

[7] The authors also consider expectations which "track quality"; a case which refers to vertical product differentiation.

[8] It is straightforward to see the correspondence between the notions of "track surplus" and "stubbornly favor one firm" with our weak and strong expectations, respectively.

We proceed as follows. In Section 2 we present the model. In Section 3 we analyze both polar cases of strong and weak expectations and the mixed case in which consumers hold partially stubborn expectations. In Section 4 we analyze firms' compatibility incentives as well as social and consumer preferences for compatibility. Finally, Section 5 concludes.

2 The Model

We analyze a duopoly where firms offer differentiated products which exhibit positive network effects. We suppose a Hotelling duopoly model with a unit mass of consumers uniformly distributed on the interval $[0, 1]$. Each consumer is indexed by $x \in [0, 1]$ indicating the ideal point in the product-characteristic space. We suppose two firms $i = A, B$ each offering one product at price p_i. Firm A's product is located at $x = 0$ and firm B's product is located at $x = 1$. Each consumer buys at most one unit of one of the two products offered in the market.

We assume that both firms produce under the same cost conditions. We set production costs to zero. Firms set their prices p_i independently and simultaneously. Consumer utility is increasing in network size with slope $b > 0$. The network size associated with product i is determined by the total number of compatible products sold in the market.[9] When both products are incompatible, the utility a consumer with address $x \in [0, 1]$ derives from product A is

$$U_x^A(p_A, \alpha_A) = v + b\alpha_A - tx - p_A, \tag{1}$$

where α_A denotes firm A's market share, v is the stand-alone value of the product, and t is the "transportation" cost rate. Accordingly, the utility from purchasing product B is given by

$$U_x^B(p_B, \alpha_B) = v + b\alpha_B - t(1 - x) - p_B, \tag{2}$$

where α_B stands for firm B's market share. We assume that the stand-alone value v is sufficiently large such that the market is always covered in equilibrium; i.e., $\alpha_A + \alpha_B = 1$ holds. Hence, the total quantity of product i sold in the market is equal to its market share, α_i. Accordingly, product i's market share, α_i, multiplied by the network effects parameter, b, gives the total network effects associated with the consumption of product i.

[9] Products of a single firm are always compatible with each other.

When firms' products are compatible, the utility from product i is given by $U(p_i, 1)$ such that network effects are always at their maximum level (we analyze compatible products in Section 4).

Consumers make their buying decisions simultaneously and independently. Consumers form expectations about firms' market shares which are the result of consumers' purchasing decisions. We consider two polar cases of expectation formation: strong expectations and weak expectations (below we also examine the "mixed" case). We model both cases of consumer expectations by adjusting the timing of our game. When expectations are weak, consumers form their expectations *after* prices have been set. In contrast, if expectations are strong, then consumers determine their expectations *before* firms set prices.

We now define the equilibrium concepts. First, consider the case of strong expectations. In the first stage of the game, consumers form expectations about each firm's market share, α_i^e, for $i = A, B$. In the second stage, firms set prices to maximize their profits. Finally, in the third stage, consumers make their purchasing decisions. Consumer demand for product i is then a function of firms' prices and consumers' (strong) expectations.[10] We write the corresponding demand function as $q(p_i, p_{j,} \alpha_i^e)$ (we derive the demand function in the next section).

We solve the strong expectations game for fulfilled expectations Nash equilibria. In such an equilibrium each firm's price maximizes its profit $\pi_i(p_i, p_j, \alpha_i^e) := p_i q(p_i, p_{j,} \alpha_i^e)$ for a given price of the rival and given consumer expectations. Moreover, consumer expectations are fulfilled; i.e., each firm's equilibrium market share is equal to the expected market share. The following definition summarizes these considerations (the superscript "SE" stands for strong expectations and indicates equilibrium values).

Definition 1 (Strong Expectations). *When consumers hold strong expectations, we solve for the fulfilled expectations Nash equilibrium which is a vector of prices and market shares* $(p_A^{SE}, p_B^{SE}, \alpha_A^{SE})$*, such that each consumer buys the product which maximizes his utility given firms' prices* p_A^{SE} *and* p_B^{SE} *and consumers' strong expectations* α_A^{SE}*. Each firm i chooses the price optimally given the demand* $q(p_i, p_{j,}^{SE} \alpha_i^{SE})$ *and firm j's price* p_j^{SE} $(i, j = A, B,\ i \neq j)$*.*

[10] We include in the demand function only consumer expectations about firm i's market share as we assumed that the market is always covered in equilibrium; i.e., $\alpha_A + \alpha_B = 1$ holds.

Moreover, consumer expectations are fulfilled. Hence, $(p_A^{SE}, p_B^{SE}, \alpha_A^{SE})$ *fulfills the conditions*

$$\begin{aligned} p_i^{SE} &= \arg\max_{p_i \geq 0} \ p_i q(p_i, p_{j,}^{SE} \alpha_i^{SE}) \text{ and} \\ \alpha_i^{SE} &= q(p_i^{SE}, p_j^{SE}, \alpha_i^{SE}), \end{aligned}$$

for $i, j = A, B$*,* $i \neq j$*.*

We use the Nash equilibrium concept to solve the weak expectations case.[11] In the first stage, firms choose simultaneously and noncooperatively their prices. In the second stage, consumers make their purchasing decisions, where each consumer chooses the product which maximizes his utility (given firms' prices and the choices of the other consumers). We denote the corresponding demand function by $q(p_i, p_j)$.[12] The following definition states the Nash equilibrium for the weak expectations case (the superscript "WE" indicates the equilibrium values).

Definition 2 (Weak Expectations). *When consumers hold weak expectations, we solve for the Nash equilibrium which is a vector of prices and market shares* $(p_A^{WE}, p_B^{WE}, \alpha_A^{WE})$*, such that each consumer buys the product which maximizes his utility given firms' prices* p_A^{WE} *and* p_B^{WE} *and the choices of the other consumers* α_A^{WE}*. Each firm* i *chooses its price optimally given the demand* $q(p_i, p_j^{WE})$ *and firm* j*'s price* p_j^{WE}*; i.e.,*

$$p_i^{WE} = \arg\max_{p_i \geq 0} \ p_i q(p_i, p_j^{WE}) \text{ for } i, j = A, B \text{ and } i \neq j.$$

Evaluating the demand at the equilibrium prices gives $\alpha_i^{WE} = q(p_i^{WE}, p_j^{WE})$*.*

We solve the mixed case where consumer expectations are partially stubborn by combining Definitions 1 and 2.

3 Analysis and Main Results

We first analyze both polar cases where consumers hold either strong expectations or weak expectations. We then turn to the setting in which consumers are partially stubborn so that expectations are a hybrid of strong and weak expectations. Finally, we compare our results.

[11] This approach was adopted in Farrell and Saloner (1992), Baake and Boom (2001), and Grilo, Shy, and Thisse (2001).

[12] Note that consumer expectations do not enter the demand function.

3.1 Strong Expectations

We assume that consumers form expectations before observing firms' prices, while firms set prices given consumer expectations. Consumers make their purchasing decisions based on firms' expected market shares and their prices. Substituting the expected market shares α_i^e into the utilities (1) and (2), it is straightforward to obtain the demand for product i as

$$q(p_i, p_j, \alpha_i^e) =$$

$$\begin{cases} 1 & \text{if} \quad p_j - p_i \geq b(1 - 2\alpha_i^e) + t \\ \frac{1}{2} + \frac{b(2\alpha_i^e - 1) - p_i + p_j}{2t} & \text{if} \quad b(1 - 2\alpha_i^e) - t \leq p_j - p_i \leq b(1 - 2\alpha_i^e) + t \\ 0 & \text{if} \quad p_j - p_i \leq b(1 - 2\alpha_i^e) - t, \end{cases} \tag{3}$$

with $i, j = A, B$ and $i \neq j$. Notice, the demand function (3) depends on the initially formed strong expectations.

For the exposition of the subsequent analysis it is convenient to define the ratio of the product differentiation parameter to the network effects parameter by $\kappa := t/b$, with $\kappa \in (0, \infty)$. The ratio κ is high (low) if product differentiation is large (low) and/or network effects are small (large).

We start with the duopoly equilibrium, in which both firms share the market. Maximization of firms' profits, $\pi_i(p_i, p_j, \alpha_i^e)$, yields the optimal prices $p_i(\alpha_i^e) = b\left[\kappa + (2\alpha_i^e - 1)/3\right]$ for given consumer expectations. Requiring that expectations are fulfilled, we obtain the equilibrium market shares $\alpha_i^{d,SE} = 1/2$ and prices $p_i^{d,SE} = b\kappa$, where the superscript "d" indicates the duopoly equilibrium. The duopoly equilibrium always exists, as each firm's maximization problem is strictly concave and $\alpha_i^{d,SE}$ and $p_i^{d,SE}$ are positive for any values of b and t.

We turn now to the monopoly equilibria, where one firm i becomes the monopolist. In this equilibrium prices are given by $p_i^{m,SE} = b(1-\kappa)$ and $p_j^{m,SE} = 0$, where the superscript "m" stands for the monopoly equilibrium. That is, firm i monopolizes the market with $p_i^{m,SE} = b(1 - \kappa)$ and firm j cannot do better than charging the lowest possible price $p_j^{m,SE} = 0$. Those prices constitute an equilibrium only if firm i does not have a unilateral incentive to increase its price, which requires

$$\left.\frac{\partial \pi_i}{\partial p_i}\right|_{p_i = b(1-\kappa),\ p_j = 0,\ \alpha_i^e = 1} \leq 0. \tag{4}$$

Evaluating Condition (4), we obtain the parameter restriction $\kappa \leq 1/3$. We summarize our results as follows.[13]

Proposition 1. *Suppose products are incompatible and consumers hold strong expectations. Then there exists a unique duopoly equilibrium, in which each firm i sets the price $p_i^{d,SE} = b\kappa$ and serves half of the market. If network effects are sufficiently large (i.e., $\kappa \leq 1/3$ holds), then two monopoly equilibria also exist, where firm i gains the entire market and sets the price $p_i^{m,SE} = b(1-\kappa)$, while the rival firm cannot do better than setting $p_j^{m,SE} = 0$ ($i, j = A, B$ and $i \neq j$).*

If network effects are relatively weak (or, conversely, products are sufficiently differentiated), then only the duopoly equilibrium exists (i.e., if $\kappa > 1/3$ holds). For larger network effects (or, rather homogeneous products, i.e., if $\kappa \leq 1/3$) two additional monopoly equilibria exist. In the monopoly equilibrium one firm gains the entire market with a "limit" price. This price makes the consumer at the other end of the Hotelling line indifferent between buying the monopolist's product (involving a disutility of t but giving rise to network utility b) or the rival firm's product which is offered at a price of zero (though lacking any network utility).

From Proposition 1 it follows that the winning firm in the monopoly equilibrium makes a higher profit than in the duopoly equilibrium.[14,15] This result reveals how crucial expectations and network effects are for firms' behavior and their performance. If consumers expect a firm to become a monopolist, that firm gains the entire market with a price which is higher than the price an undercutting firm would have to charge in order to gain the entire market if consumers are expecting the duopoly outcome.[16]

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[13] In the following we ignore the non-generic case $\kappa = 1$.

[14] Comparison of the duopoly profit, $\pi^{d,SE} = b\kappa/2$, with the monopoly profit, $\pi^{m,SE} = b(1-\kappa)$, gives $\pi^{m,SE} > \pi^{d,SE}$ for all $k \leq 1/3$ (which is the parameter region where both equilibria exist).

[15] It is noteworthy that the reasoning of d'Aspremont, Gabszewicz, and Thisse (1979) regarding the non-existence of an (interior) duopoly equilibrium does not apply to our model with network effects. That paper shows within a linear Hotelling model that an equilibrium does not exist when the monopolization of the market leads to a higher profit than the profit in the (interior) duopoly equilibrium.

[16] This reasoning is not specific to the case of strong expectations. Below we show that it carries over to the weak expectations case. However, as consumers do not revise their expectations to actually charged prices the effect is more pronounced under strong expectations. As a consequence, the parameter region where the monopoly

3.2 Weak Expectations

We solve the case of weak expectations for standard Nash equilibria: every consumer conditions his purchasing decisions on firms' prices and the decisions of the other consumers which give rise to market shares α_A and α_B for products A and B, respectively. Hence, the utility of a consumer located at x from purchasing good i is given by (1) if he buys product A and by (2) if he buys product B. Each consumer chooses in the second stage the product that yields the highest utility for him, given prices and the other consumers' choices (i.e., α_A and α_B). As each consumer is infinitesimal, his own choice does not affect market shares. The equilibrium demand $q(p_A, p_B)$ is then derived from noticing that the consumer with address $x = q(p_A, p_B)$ is indifferent between product A and product B, when prices are p_A and p_B and the market share of product A is equal to $\alpha_A = q(p_A, p_B)$.

In the following we distinguish between two cases: *i)* small network effects with $\kappa > 1$ and *ii)* large networks effects with $\kappa < 1$. Solving for the Nash equilibria we derive the following result.

Proposition 2. *Suppose products are incompatible and consumers hold weak expectations. If network effects are large (or, alternatively, product differentiation is small) (i.e., $\kappa < 1$ holds), multiple equilibria exist:*

i) a continuum of monopoly equilibria with $0 \leq p_i^{m,WE} \leq b(1-\kappa)$, $p_j^{m,WE} = 0$ and $\alpha_i^{m,WE} = 1$, and

ii) a unique duopoly equilibrium with $p_i^{d,WE} = 0$ and $\alpha_i^{d,WE} = 1/2$, for $i, j = A, B$ and $i \neq j$.

If network effects are small or, alternatively, product differentiation is large (i.e., $\kappa > 1$ holds), then a unique (duopoly) equilibrium exists, in which each firm i sets the price $p_i^{d,WE} = b(\kappa - 1)$ and obtains half of the market.

Proof. Assume first $\kappa > 1$. Comparing consumers' utilities (1) and (2) we obtain the demand function[17]

$$q(p_A, p_B) =$$

equilibria exist is smaller under strong expectations than under weak expectations.

[17] Assuming that prices differ not too much, we obtain the intermediate interval by equating (1) and (2) and setting firm A's market share equal to the address of the indifferent consumer.

$$\begin{cases} 1 & \text{if} \quad p_B - p_A \geq -(b-t) \\ \frac{1}{2} + \frac{p_A - p_B}{2(b-t)} & \text{if} \quad b - t \leq p_B - p_A \leq -(b-t) \\ 0 & \text{if} \quad p_B - p_A \leq b - t. \end{cases} \tag{5}$$

Using symmetry yields firm B's demand. We start with the duopoly equilibrium. Maximization of firms' profits yields the first-order conditions $(b-t) + 2p_i - p_j = 0$ $(i, j = A, B,\ i \neq j)$, which give rise to equilibrium prices $p_i^{d,WE} = b(\kappa - 1)$ and market shares $\alpha_i^{d,WE} = 1/2$. Since $\kappa > 1$, firms' profits are maximized at $p_i^{d,WE} = b(\kappa - 1)$. We now rule out monopoly equilibria (we indicate candidate equilibrium values with an asterisk). In the monopoly equilibrium with $\alpha_i^* = 1$, prices must fulfill $p_j^* - p_i^* \geq -(b-t)$ from which $p_j^* \geq p_i^* + (t-b) > 0$ follows. The latter condition implies that firm j must set a positive price. Hence, it always has an opportunity to profitably undercut and there can be no monopoly Nash equilibrium.

Assume now $\kappa < 1$. The demand for firm i's product then becomes

$$q(p_i, p_j) =$$

$$\begin{cases} 1 & \text{if} \quad p_j - p_i \geq -(b-t) \\ \frac{1}{2} + \frac{p_i - p_j}{2(b-t)} & \text{if} \quad -(b-t) \leq p_j - p_i \leq b - t \\ 0 & \text{if} \quad p_j - p_i \leq b - t. \end{cases} \tag{6}$$

Note that for prices in the interval $-(b-t) \leq p_j - p_i \leq b - t$, the demand can take three different values, which implies that $q(p_i, p_j)$ is a demand correspondence. Note also that the demand in the intermediate interval, $1/2 + (p_i - p_j)/[2(b-t)]$, is increasing in p_i. With large network effects multiple Nash equilibria are possible. At this point we refer to GST who showed in their setting that the invariance axiom implies that one firm must set a price of zero in the monopoly equilibrium, while both firms set their prices equal to zero in the duopoly equilibrium. We, therefore, obtain the types of equilibria stated in parts *i)* and *ii)* of the proposition.[18]

We illustrate the invariance axiom by showing that there exist no equilibria with $p_i^*, p_j^* > 0$. Given a pair of prices and the associated market equilibrium, the invariance axiom requires the following: If both firms increase their prices by the same amount, then the new prices must induce an equilibrium in which firms' market shares remain constant (provided that the market

[18] The multiplicity of equilibria in the monopoly outcome depends on the assumption that all consumers would switch to the rival firm if the monopolist increases its price even slightly.

is covered). First note that equilibrium prices must satisfy $-(b-t) \leq p_i^* - p_j^* \leq b-t$. Indeed, assume that $p_i^* - p_j^* > b-t$, in which case $\alpha_i = 0$, so that firm i always finds it optimal to decrease its price to get a positive market share. If $p_i^* - p_j^* < -(b-t)$, then firm j has the same incentive to deviate. Hence, the only case which remains is $-(b-t) \leq p_i^* - p_j^* \leq b-t$. In that case demand for firm i can take three different values: $\alpha_i = 0$, $\alpha_i = 1$ and $\alpha_i = \alpha_i^I := (1/2) + (p_i^* - p_j^*)/[2b(1-\kappa)]$. Assume first that $p_i^*, p_j^* > 0$ and α_i^I constitute an equilibrium outcome. We show that such an equilibrium does not satisfy the invariance axiom. For $p_i^*, p_j^* > 0$ and α_i^I to be an equilibrium any price pair (p_i, p_j^*) with $p_i > p_i^*$ must induce $\alpha_i = 0$, as otherwise firm i can increase both its price and quantity (note that α_i^I increases in p_i). It then follows from the invariance axiom that any price pair above the line $p_i = p_i^* - p_j^* + p_j$ (the line with slope 1 passing through the point (p_i^*, p_j^*)) must also induce $\alpha_i = 0$. But then firm j can increase its profit by slightly lowering its price (given that $p_i = p_i^*$, only price pairs (p_i, p_j) with $p_j < p_j^*$ lie in the region where $\alpha_i = 0$). Hence, $p_i^*, p_j^* > 0$ and α_i^I cannot constitute an equilibrium outcome under the invariance axiom. Assume now that $p_i^*, p_j^* > 0$ and $\alpha_i = 1$ describe an equilibrium outcome. Then any price pair (p_i^*, p_j) with $p_j \neq p_j^*$ must induce $\alpha_i = 1$, as otherwise firm j can increase its profit by unilaterally changing its price. The invariance axiom implies then that any price pair above the line $p_i = p_i^* - p_j^* + p_j$ must also induce $\alpha_i = 1$. But then, firm i can instead increase its profit by increasing its price. We can use a similar argument to prove that $p_i^*, p_j^* > 0$ and $\alpha_i = 0$ cannot constitute an equilibrium. *Q.E.D.*

Proposition 2 states that multiple Nash equilibria exist when network effects are large. In that case duopoly and monopoly equilibria are possible. In the duopoly equilibrium both firms compete away all profits and set prices equal to zero. In the monopoly equilibria the monopolist may set a price up to $b(1-\kappa)$, while the competitor cannot do better than setting always its price to zero.

3.3 Mixed Expectations

We now assume that each consumer neither holds purely strong nor entirely weak expectations. Instead, we suppose that consumer expectations are a hybrid of weak expectations and strong expectations weighted by λ and $1-\lambda$, respectively. We interpret $1-\lambda$ as the degree of stubbornness of consumer expectations. That is, with probability $1-\lambda$ a consumer forms his expectations before firms set prices and with counter probability λ his expectations are adjusted *after* firms have set prices. All consumers are homogeneous with regard to the degree of stubbornness.[19]

We can integrate the degree of stubbornness into the formulation of consumers' utilities (1) and (2). The utility of a consumer with address x from purchasing good A can then be written as

$$U_x^A(p_A, p_B, \alpha_A^e, \lambda) = v + \lambda b\alpha_A + (1-\lambda)b\alpha_A^e - tx - p_A \tag{7}$$

and, accordingly, the utility from buying product B as

$$U_x^B(p_A, p_B, \alpha_B^e, \lambda) = v + \lambda b\alpha_B + (1-\lambda)b\alpha_B^e - t(1-x) - p_B. \tag{8}$$

As consumers' purchasing decisions are influenced by strong expectations, α_i^e, a firm's demand in the mixed case, $q(p_i, p_j, \alpha_i^e, \lambda)$, is a function of firms' prices, consumers' strong expectations, and the degree of stubbornness. Firms' demands must be such that a consumer with address $x = q(p_A, p_B, \alpha_A^e, \lambda)$ is indifferent between products A and B when firm A's market share is $\alpha_A = q(p_A, p_B, \alpha_A^e, \lambda)$, firms' prices are p_A and p_B, and with probability $1-\lambda$ consumers expect firm A (B) to hold a market share α_A^e $(1-\alpha_A^e)$.

We solve for the fulfilled expectations Nash equilibria such that stubborn expectations, α_A^e, are fulfilled in equilibrium. In the following we distinguish between two cases: *i)* small network effects with $\kappa > \lambda$ and *ii)* large networks effects with $\kappa < \lambda$.[20]

Assume first that network effects are small ($\kappa > \lambda$). Comparing the utilities (7) and (8) yields firm i's demand

$$q(p_i, p_j, \alpha_i^e, \lambda) =$$

$$\begin{cases} 1 & \text{if} \quad p_j - p_i \geq b\phi(\alpha_i^e, \lambda) + t \\ \frac{1}{2} + \frac{(1-\lambda)b(2\alpha_i^e - 1)}{2(t-\lambda b)} + \frac{p_j - p_i}{2(t-\lambda b)} & \text{if} \quad b\zeta(\alpha_i^e, \lambda) - t \leq p_j - p_i \leq b\phi(\alpha_i^e, \lambda) + t \\ 0 & \text{if} \quad p_j - p_i \leq b\zeta(\alpha_i^e, \lambda) - t, \end{cases} \tag{9}$$

[19] In the Appendix we present an alternative interpretation of the mixed expectations case which gives rise to the same results (we thank an anonymous referee for that suggestion). Precisely, suppose two types of consumers, one holding strong expectations and the other one holding weak expectations. Consumers of both types are uniformly distributed on the unit interval. The mass of consumers with weak expectations is λ, while that of consumers with strong expectations is $1-\lambda$. Again, we can then interpret $1-\lambda$ as measuring the degree of stubbornness in the market.

[20] We do not consider the non-generic case $\kappa = \lambda$.

with $\phi(\alpha_i^e, \lambda) := 2(1-\lambda)(1-\alpha_i^e) - 1$ and $\zeta(\alpha_i^e, \lambda) := 1 - 2(1-\lambda)\alpha_i^e$. We start with the duopoly equilibrium. Maximization of firms' profits yields prices $p_i(\alpha_i^e) = t - b\lambda + (1-\lambda)b(2\alpha_i^e - 1)/3$. Imposing the fulfilled Nash equilibrium condition that consumers' strong expectations are fulfilled, we obtain the equilibrium prices and market shares given by $p_i^{d,ME} = b(\kappa - \lambda)$ and $\alpha_i^{d,ME} = 1/2$, respectively (the superscript "ME" stands for mixed expectations). Those market shares and prices constitute an equilibrium as each firm's maximization problem is strictly concave and $b(\kappa - \lambda) > 0$ holds by assuming small network effects.

We now turn to the monopoly equilibria where one firm i ($i = A, B$) becomes the monopolist. In such an equilibrium it must hold that $\alpha_i^e = 1$ and $p_j = 0$ ($i \neq j$) as otherwise firm j can profitably undercut. Hence, p_i must satisfy $p_i \leq b(1-\kappa)$, which implies $\kappa \leq 1$. Moreover, firm i must not have an incentive to increase its price, which requires $\kappa \leq (1+2\lambda)/3$. The latter condition is more restrictive than $\kappa \leq 1$. Hence, if $\lambda < \kappa \leq (1+2\lambda)/3$, then two monopoly equilibria exist.

Assume next that $0 < \kappa < \lambda$, so that network effects are large. Firm i's demand is then given by

$$q(p_i, p_j, \alpha_i^e, \lambda) =$$
$$\begin{cases} 1 & \text{if} \quad p_j - p_i \geq b\phi(\alpha_i^e, \lambda) + t \\ \frac{1}{2} + \frac{(1-\lambda)b(2\alpha_i^e - 1)}{2(t-\lambda b)} + \frac{p_j - p_i}{2(t-\lambda b)} & \text{if} \quad b\phi(\alpha_i^e, \lambda) + t \leq p_j - p_i \leq b\zeta(\alpha_i^e, \lambda) - t \\ 0 & \text{if} \quad p_j - p_i \leq b\zeta(\alpha_i^e, \lambda) - t. \end{cases} \tag{10}$$

Note that firm i's demand is a correspondence for any $\alpha_i^e \in [0,1]$ as it can take three possible values in the intermediate interval. Indeed, requiring that $b\zeta(\alpha_i^e, \lambda) - t > b\phi(\alpha_i^e, \lambda) + t$, we obtain the condition $\kappa < \lambda$, which holds by assuming large network effects. In equilibrium, either one firm gains the whole market or shares it with the rival. Moreover, expectations must be fulfilled. We first consider the case where firm i gains the whole market with $\alpha_i^e = \alpha_i^{m,ME} = 1$.[21] The monopoly outcome $q(p_i, p_j, 1, \lambda) = 1$ can only constitute an equilibrium if firm j sets a price of zero, while firm i sets any non-negative price up to the "limit" price, which is $p_i = -b\left[2(1-\lambda)(1-\alpha_i^e) - 1\right]|_{\alpha_i^e = 1} - t = b(1-\kappa)$. Hence, we get the monopoly equilibria with $\alpha_i^{m,ME} = 1$ and prices $0 \leq p_i^{m,ME} \leq b(1-\kappa)$ and $p_j^{ME} = 0$, for $i, j = A, B$ and $i \neq j$.

We next consider the duopoly case with $0 < \alpha_i^e = \alpha_i^{d,ME} < 1$. The duopoly outcome, with

[21] In the following we apply again the invariance axiom as used in GST (see proof of Proposition 2).

$0 < q(p_i, p_j, \alpha_i^{d,ME}, \lambda) < 1$, can only constitute an equilibrium if $p_i^{d,ME} = 0$ holds, for $i = A, B$. As expectations must be fulfilled, we obtain from the demand (10) the requirement

$$\alpha_i^{d,ME} = \frac{1}{2} + \frac{(1-\lambda)b(2\alpha_i^{d,ME} - 1)}{2(t - \lambda b)},$$

which yields equal market sharing, with $\alpha_A^{d,ME} = \alpha_B^{d,ME} = 1/2$. We summarize our results in the following proposition.

Proposition 3. *Assume incompatible products and mixed expectations. Depending on λ and κ the following equilibria emerge:*

i) If $\kappa < \lambda$, then there exists a unique duopoly equilibrium with $\alpha_i^{d,ME} = 1/2$ and $p_i^{d,ME} = 0$. There also exists a continuum of monopoly equilibria with $\alpha_i^{m,ME} = 1$ and prices $p_i^{m,ME} \in [0, b(1-\kappa)]$ and $p_j^{m,ME} = 0$ ($i, j = A, B$, $i \neq j$).

ii) If $\lambda < \kappa \leq (1 + 2\lambda)/3$, then there exists a unique duopoly equilibrium with $\alpha_i^{d,ME} = 1/2$ and $p_i^{d,ME} = b(\kappa - \lambda)$. There are also two monopoly equilibria with $\alpha_i^{m,ME} = 1$ and prices $p_i^{m,ME} = b(1-\kappa)$ and $p_j^{m,ME} = 0$.

iii) If $\kappa > (1 + 2\lambda)/3$, then there exists a unique (duopoly) equilibrium with $\alpha_i^{d,ME} = 1/2$ and prices $p_i^{d,ME} = b(\kappa - \lambda)$.

From Proposition 3 we observe that both polar cases of strong and weak expectations recur as special cases under mixed expectations. If $\lambda = 0$, parts *ii)* and *iii)* of Proposition 3 reiterate the message of Proposition 1.[22] If $\lambda = 1$, parts *i)* and *iii)* of Proposition 3 state the same equilibria as Proposition 2.[23]

We infer from Proposition 3 that multiple equilibria become less likely when stubbornness (i.e., $1 - \lambda$) increases. An increase in $1 - \lambda$ makes the parameter range which yields a unique equilibrium ($\kappa > (1 + 2\lambda)/3$) as well as the parameter range with a duopoly and two monopoly equilibria ($\lambda < \kappa \leq (1 + 2\lambda)/3$) larger. Conversely, the interval with infinitely many equilibria ($\kappa < \lambda$) becomes smaller with increasing stubbornness. Moreover, the impact of a change in λ is more pronounced the larger network effects become. If network effects are sufficiently small (with $\kappa > 1$ holding), a unique equilibrium emerges for any value of λ.

[22] Part *i)* of Proposition 3 is irrelevant when $\lambda = 0$ since $\kappa < 0$ is not admissible.

[23] If $\lambda = 1$, then case *ii)* of Proposition 3 is irrelevant since there can be no κ satisfying $1 < \kappa \leq 1$.

It also follows from Proposition 3 that equilibrium prices cannot increase when expectations become less stubborn (i.e., λ increases). This result is straightforward for the monopoly equilibria and the duopoly equilibrium when network effects are large ($\kappa < \lambda$), where prices do not depend on λ. Given that network effects are sufficiently large ($\kappa > \lambda$), duopoly equilibrium prices, $p_i^{d,ME} = b(\kappa - \lambda)$, monotonically decrease in λ. Moreover, prices decrease faster when network effects are large. The relative change in prices is also larger, the smaller product differentiation becomes.

To understand these relationships, we consider the derivative of the demand function (9) in the intermediate interval with respect to p_i which gives

$$\frac{\partial q_i(p_i, p_j, \alpha_i^e, \lambda)}{\partial p_i} = -\frac{1}{2b(\kappa - \lambda)}. \tag{11}$$

The right-hand side of (11) is decreasing in λ provided that $\lambda < \kappa$. Hence, if the indifferent consumers is located in the open interval $(0, 1)$, the demand function becomes more price-sensitive when expectations become weaker (i.e., λ increases). Consumers with relatively weak expectations are more eager to take into account the positive demand effect of a price reduction. That in turn, intensifies price competition leading to lower prices in equilibrium.

Analyzing the dependence of equilibrium prices on the ratio κ, we observe that in the duopoly equilibrium prices decrease when network effects become larger or/and product differentiation becomes less intense. When $\kappa > \lambda$, the prices $p_i^{d,ME} = b(\kappa - \lambda)$ monotonically decrease in b. If $\kappa < \lambda$, the equilibrium prices in the duopoly equilibrium do not respond to changes in b and remain at the lowest possible level of zero. The intuition for this result can be again inferred from inspecting the derivative (11). For a given level of stubbornness, consumers react more to changes in firms' prices when network effects are large.[24]

However, the impact of network effects on the monopolist's price in the monopoly equilibrium is ambiguous. When $\kappa < \lambda$, the monopolist can set any non-negative price up to $p_i^{m,ME} = b(1 - \kappa)$, while with lower network effects (when $\kappa > \lambda$ holds), the price is always at the level $b(1 - \kappa)$. Yet, the price $p_i^{m,ME} = b(1 - \kappa)$ increases in b.

Taking the level of prices in the duopoly equilibrium as an inverse measure of competition, the following corollary follows directly.

[24] Precisely, using (11) we obtain $\partial(|\partial q_i(\cdot)/\partial p_i|)/\partial b = 2\lambda/\left[2b(\kappa - \lambda)\right]^2 > 0$.

Corollary 1. *If the degree of stubbornness and/or network effects increase, then the intensity of competition increases as well.*

3.4 Comparison of Results

We now turn to the comparison of social welfare and consumer surplus under the two types of equilibria (duopoly and monopoly). We use our results for the mixed expectations case which nests the weak and strong expectations scenarios as special cases. In the following we drop the superscript indicating the mixed expectations case. Consumer surplus and social welfare in the duopoly equilibrium are given by $CS^d = v + b(1/2 - \kappa/4) - p^d$ and $SW^d = v + b(1/2 - \kappa/4)$. For the monopoly equilibrium we obtain the corresponding values $CS^m = v + b(1 - \kappa/2) - p^m$ and $SW^m = v + b(1 - \kappa/2)$, respectively. Table 1 summarizes for the duopoly and monopoly outcomes equilibrium values of prices, consumer surplus, profits, and social welfare.

	$\kappa > \lambda$ (small network effects)		$\kappa < \lambda$ (large network effects)	
Equilibrium	duopoly	monopoly*	duopoly	monopoly
Price	$b(\kappa - \lambda)$	$b(1 - \kappa)$	0	$[0, b(1 - \kappa)]$
CS	$v + b\left(\frac{1}{2} - \frac{5\kappa}{4} + \lambda\right)$	$v + b\frac{\kappa}{2}$	$v + b\left(\frac{1}{2} - \frac{\kappa}{4}\right)$	$\left[v + b\frac{\kappa}{2}, v + b\left(1 - \frac{\kappa}{2}\right)\right]$
Profits	$b(\kappa - \lambda)$	$b(1 - \kappa)$	0	$[0, b(1 - \kappa)]$
SW	$v + b\left(\frac{1}{2} - \frac{\kappa}{4}\right)$	$v + b\left(1 - \frac{\kappa}{2}\right)$	$v + b\left(\frac{1}{2} - \frac{\kappa}{4}\right)$	$v + b\left(1 - \frac{\kappa}{2}\right)$

*For $\lambda < \kappa$, the monopoly equilibrium exists if and only if $\lambda \geq (3\kappa - 1)/2$ and $\kappa < 1$.

Table 1. Duopoly and monopoly equilibria under incompatibility

Corollary 2 states social and consumer preferences for different equilibrium constellations (holding parameter values fixed).

Corollary 2. *Assume incompatible products and suppose that both duopoly and monopoly equilibria exist. Social welfare is higher in any monopoly equilibrium when compared with the duopoly equilibrium. Consumer preferences with respect to the type of the equilibrium are as follows:*
i) Suppose network effects are relatively small (i.e., $\kappa > \lambda$ holds). Then there exists a critical value $\lambda' := (7\kappa - 2)/4$ for all $\kappa < 2/3$ such that $CS^d \geq CS^m$ holds for all $\lambda \geq \lambda'$ (with equality holding at $\lambda' = \lambda$), while in the remaining parameter region $CS^d < CS^m$ holds.

ii) Suppose network effects are relatively large (i.e., $\kappa < \lambda$). If the monopolist sets the lowest price of zero, then $CS^d < CS^m$. If the monopolist sets the highest possible price $b(1-\kappa)$, then $CS^d > CS^m$ if $\kappa < 2/3$. If $\kappa \geq 2/3$, then $CS^d \leq CS^m$ (with equality holding at $\kappa = 2/3$).

Corollary 2 states that social welfare is always maximized in the monopoly equilibrium; i.e., when the market mechanism achieves de facto standardization. The result becomes intuitive, if we recall from Proposition 3 that the monopoly equilibrium only emerges when product differentiation is relatively small or network effects are large enough (i.e., when κ fulfills $\lambda < \kappa \leq (1+2\lambda)/3$ or $\kappa < \lambda$). Those restrictions make the monopoly equilibrium socially attractive as the costs of less product variety are kept small relative to the benefits from maximum network effects.

Consumer preferences for the type of equilibrium critically depend on the prevailing prices which mirror the underlying fundamentals. Consider first the case of relatively small network effects (see part *i)* of Corollary 2, where $\kappa > \lambda$ holds). The price in the duopoly equilibrium decreases when expectations become more price-sensitive leaving more of the network benefits to consumers. That sharply contrasts with the monopoly equilibrium, which allows the monopolist to extract all of the network benefits with a price of $b(1-\kappa)$. Hence, in the assumed parameter region, consumers are more likely to prefer the duopoly equilibrium when both network effects and the price-sensitivity of demand are sufficiently large. Hence, it is necessary that expectations are sufficiently weak (i.e., price-sensitive) such that consumers find the duopoly equilibrium more attractive than the monopoly outcome.

Moreover, increasing the significance of product differentiation, t, makes the monopoly equilibrium more attractive for consumers as the monopolist reduces its price by t to keep the rival firm out. Hence, if product differentiation becomes large enough, consumer preferences are aligned with social preferences.

Consider, finally, the case when network effects are large (see part *ii)* of Corollary 2, where $\kappa < \lambda$ holds). In the duopoly equilibrium consumers enjoy zero prices, while the monopolist may be able to set a strictly positive price. Clearly, consumers then favor the monopoly equilibrium when the monopolist sets the most competitive price. If, to the contrary, the least competitive price prevails in the monopoly equilibrium, then consumer preferences, again, depend on the significance of product differentiation. When products are sufficiently differentiated ($\kappa \geq 2/3$), then consumers are better off in the monopoly equilibrium, in which the monopolist extracts all the network effects but reduces his price by t to drive out the rival firm. When product differentiation is relatively small ($\kappa < 2/3$), then the conflict between consumer surplus and social welfare emerges again.

Comparing social preferences for de facto standardization with consumer preferences, we can identify instances of alignment and conflict. Social and consumer preferences are most likely to be aligned when product differentiation is large enough or network effects play only a minor role (i.e., $\kappa \geq 2/3$ holds). In particular, if the monopolist sets the highest possible price (see part *ii)* of Corollary 2), then any potential for conflict vanishes (given $\kappa \geq 2/3$ holds).

If, however, network effects become more important or product differentiation less pronounced (such that $\kappa < 2/3$ becomes true), we observe that the conflict between a social welfare and a consumer surplus standard becomes more likely the less stubborn consumer expectations become. Part *ii)* of Corollary 2 shows that the conflict is ubiquitous when expectations are highly price-sensitive (i.e., $\lambda > \kappa$) and the least competitive monopoly equilibrium emerges. Similarly, Part *i)* reveals that consumers favor a duopoly equilibrium even for relatively stubborn expectations (i.e., $\lambda < \kappa$ holds) when product differentiation is small enough relative to network effects (i.e., $\kappa < 2/3$ holds).

Our result that consumers may prefer a socially inefficient market outcome is related to Farrell and Saloner (1992) who argued that the existence of (imperfect) converters makes a standardization outcome less likely, so that overall incompatibilities tend to be larger with converters. They interpret their finding as an inefficiency due to the *irresponsibility of competition*. In those instances, "[i]t might be better if some good were not offered at all, or were offered only at a high price, because consumers use it 'irresponsibly'; but with competition, no agent can decide that a good will not be offered, or that its price shall be high" (Farrell and Saloner 1992, p. 13).

4 Compatibility: Incentives and Welfare

We now allow for the option to make the products of both firms compatible. The utility from consuming the product of firm $i = A, B$ is then given by $U_x^i(p_i, 1)$. When products are compatible, the amount of network effects provided by any firm is fixed at b. Hence, consumer expectations are irrelevant for their purchasing choices. The following proposition states the Nash equilibrium when products are compatible (equilibrium values are marked by "c").

Proposition 4. *Suppose products are compatible. Then a unique (duopoly) equilibrium emerges in which each firm sets the price $p_i^c = b\kappa$ $(i = A, B)$, serves half of the market and realizes the profit $\pi_i^c = bk/2$. Consumer surplus and social welfare are given by $CS^c = v + b(1 - 5\kappa/4)$ and $SW^c = v + b(1 - \kappa/4)$, respectively.*

Proof. We consider first the duopoly equilibrium, where each firm i maximizes its profit $[1/2 + (p_j - p_i)/2t]p_i$ for a given price of the competitor $(i, j = A, B,\ i \neq j)$. Maximization of firms' profits yields prices $p_i^c = b\kappa$ $(i = A, B)$ and equal market shares. Consumer surplus and social welfare are given by $CS^c = v + b(1 - 5\kappa/4)$ and $SW^c = v + b(1 - \kappa/4)$, respectively.

We prove now that there are no monopoly equilibria under compatibility. Assume, to the contrary, that firm A holds a monopoly position. Then it must hold that $U_{x=1}^A(p_A, 1) = U_{x=1}^B(p_B, 1)$ for A to gain the whole market. It must also hold that $p_B = 0$ as otherwise firm B could profitably undercut. Combining the two equalities we get $p_A = -t$; an outcome obviously not admissible. *Q.E.D.*

Comparing Proposition 4 with our previous results in Section 3, we obtain the following corollary regarding firms' compatibility incentives.

Corollary 3. *If under incompatibility the duopoly equilibrium emerges, then firms prefer compatibility for any $0 \leq \lambda \leq 1$. If the monopoly equilibrium emerges in which the monopolist sets a price of zero, then again, firms have strict incentives for compatibility. If in the monopoly equilibrium the monopolist sets the highest possible price, then firms' incentives for compatibility depend on κ as well as on the fact whether side payments are feasible or not:*
i) with side payments, firms have incentives for compatibility if $\kappa \geq 1/2$,
ii) without side payments, firms have incentives for compatibility if $\kappa \geq 2/3$.

If under compatibility the duopoly equilibrium emerges, then firms always have incentives for compatibility. The reason lies in the intensity of price competition. Under compatibility the amount of network effects (b) associated with each product is fixed and, therefore, independent of firms' pricing decisions. Under incompatibility, by decreasing its prices a firm also increases the amount of its network effects. Hence, under incompatibility demand is more price-elastic which intensifies competition. This result is in line with Shy (2001, p. 31) who states that competition is relaxed under compatibility "since under compatibility firms' network size becomes irrelevant to consumers' choice of which brand to buy."

Interestingly, Shy rules out monopoly equilibria under incompatibility. Ruling out monopoly equilibria requires to restrict the analysis to instances where network effects are relatively small (when compared with the significance of product differentiation). Yet, our comparison reveals that those instances are critical. Exactly for large network effects common wisdom may be overturned. Precisely, if the monopolist sets the highest possible price of $b(1-\kappa)$, then incentives for compatibility disappear. The intuition for a cut-off level can be inferred simply by noting that the monopolist's price $b(1-\kappa)$ decreases while the price under compatibility increases when product differentiation becomes more pronounced. Interestingly, Corollary 3 also reveals that the possibility of side-payments does not affect the finding qualitatively that firms may prefer incompatible products.

If, however, the monopolist sets the most competitive price of zero in the monopoly equilibrium under incompatibility, then we are left with the well-known conclusion that firms have strict incentives for compatibility.

Finally, we analyze whether social and consumer preferences for compatibility are aligned or whether they contradict each other. We state our results as follows.

Corollary 4. *Social welfare is always highest under compatibility when compared with the duopoly equilibrium and the monopoly equilibria under incompatibility. Consumer preferences for compatibility are as follows:*
i) Suppose network effects are relatively small (i.e., $\kappa > \lambda$ holds). Then $CS^d \geq CS^c$ if $\lambda \geq 1/2$ (with equality holding at $\lambda = 1/2$), while $CS^d < CS^c$ holds, if $\lambda < 1/2$. Moreover, $CS^m \geq CS^c$ holds, if $\kappa \geq 4/7$ (with equality holding at $\kappa = 4/7$), while $CS^m < CS^c$ holds, if $\kappa < 4/7$.
ii) Suppose network effects are relatively large (i.e., $\kappa < \lambda$ holds). Then $CS^d \geq CS^c$, if $\kappa \geq 1/2$ (with equality holding at $\kappa = 1/2$), while $CS^d < CS^c$, if $\kappa < 1/2$. Moreover, if the monopolist sets the lowest possible price, then $CS^m > CS^c$ for any λ and κ. If the monopolist sets the highest possible price, then $CS^m \geq CS^c$, if $\kappa \geq 4/7$ (with equality holding at $\kappa = 4/7$), while $CS^m < CS^c$, if $\kappa < 4/7$.

Social welfare is always highest under compatibility which maximizes network effects. As both firms share the market under compatibility, there is also no reduction in product variety. Consumer preferences for compatibility depend on expectations, product differentiation, and network effects. If network effects are relatively small (i.e., $\kappa > \lambda$ holds), then consumers enjoy lower prices in the duopoly equilibrium for sufficiently price-sensitive expectations. With $\lambda \geq 1/2$ consumers are better-off in the duopoly equilibrium compared to compatibility. Interestingly, at $\lambda = 1/2$ the negative effect of lower network effects and the positive effect of lower prices are exactly balanced, which yields indifference. Consumers also prefer the monopoly equilibrium compared to compatibility when product differentiation is strong enough relative to network effects. In the monopoly equilibrium the monopolist decreases his price by t to monopolize the market and consumer surplus in the monopoly equilibrium increases in κ. In contrast, consumer surplus under compatibility decreases in κ.

When network effects are relatively large (i.e., $\kappa < \lambda$ holds), then prices under incompatibility do not depend on λ anymore. When the monopolist sets a price of zero, consumers prefer the monopoly equilibrium when compared with compatibility. If the monopolist sets the highest possible price of $b(1-\kappa)$, then the comparison is same as in the case with a low value of λ. When product differentiation is significant, consumers prefer the duopoly equilibrium where they enjoy prices of zero while under compatibility they must pay a price of $b\kappa$.

The alignment of consumer and social preferences for compatibility, therefore, depends both on λ and κ. When λ and κ are high enough, consumers enjoy low enough prices under incompatibility which may make both the duopoly and monopoly equilibria more attractive than the equilibrium under compatibility. Hence, a conflict between consumer and social preferences for compatibility may then follow.

5 Conclusions

We examined a duopoly model where firms offer (horizontally) differentiated products that exhibit positive network effects. We focused on the role expectations play as a determinant of market conduct and market performance. Consumers form expectations about firms' market shares and may hold weak, strong, or mixed expectations. We assumed that strong (or, stubborn) expectations are determined before firms compete in prices. Under weak expectations consumers fully take account of firms' pricing decisions, while mixed expectations combine both properties.

We showed that the formation of expectations has a real impact on competition and market performance. A key insight of our analysis is that more price-sensitive (or, equivalently, less stubborn) expectations induce more competitive pricing and, with that, lower prices. The impact of consumer expectations on competition is reinforced when network effects increase and/or product differentiation is reduced. That finding has important consequences for market behavior and market outcomes in markets with network effects as well as for the desirability of de facto standardization and compatibility. More price-sensitive expectations tend to evoke a conflict between social and consumer preferences with regard to de facto standardization (i.e., the choice between the duopoly and the monopoly equilibria). Similarly, less stubborn expectations make a conflict between consumer and social preferences for compatibility more likely. Both types of conflicts tend to become more pronounced when product differentiation vanishes or network effects increase.

There are many routes for further research. One way could be to introduce vertical product differentiation and to analyze firms' incentives to raise product quality depending on consumer expectations. As expectations (weak, strong, or mixed) affect both the marginal profitability of investments and the level of profits, interesting trade-offs may emerge. Finally, another route may be to learn more about the behavioral causes which drive the degree of consumer stubbornness.

Appendix

In this Appendix we provide an alternative interpretation of the mixed expectations specification we presented in Section 3. We now assume that there are two types of consumers with opposite forms of expectations. One type only holds strong expectations and the other type has weak expectations. Consumers of both types are uniformly distributed on the unit interval. The mass of consumers with weak expectations is λ and that of consumers with strong expectations is $1-\lambda$. Again, we can interpret $1-\lambda$ as measuring the degree of stubbornness in the market.

The next proposition shows that Proposition 3 remains valid under this alternative specification (we restrict attention to parameters $\kappa > \lambda$).

Proposition A1. *Assume that consumers are of two different types, one type of mass λ holding weak expectations and the other type of mass $1-\lambda$ holding strong expectations. Suppose also $\kappa > \lambda$. Depending on λ and κ exactly the same equilibria emerge as stated in parts ii) and iii) of Proposition 3.*

Proof. We first derive the total demand, $q(\cdot)$, which is the sum of the demands of the weak and the strong expectation types; i.e., $q(\cdot) = q^{\sigma}(\cdot) + q^{\omega}(\cdot)$, where we use the superscripts "σ" and "ω" for indicating the strong and the weak expectation types, respectively. Consider first consumers with strong expectations. If prices are not too different, then we can define a marginal consumer with address $x^{\sigma}(p_A, p_B, \alpha_A^e)$ who is indifferent between both firms' products; namely

$$x^{\sigma}(p_A, p_B, \alpha_A^e) = \frac{1}{2} + \frac{b(2\alpha_A^e - 1)}{2t} + \frac{p_B - p_A}{2t}.$$

On the interval $x \in [0, x^{\sigma}(p_A, p_B, \alpha_A^e)]$ the share of consumers with strong expectations is $1-\lambda$, hence, the demand for firm A's products from strong-type consumers is $(1-\lambda)x^{\sigma}(p_A, p_B, \alpha_A^e)$, while that for firm B's products is $(1-\lambda)x^{\sigma}(p_B, p_A, \alpha_B^e)$ (again, whenever prices are not too different). Inspecting all possible price constellations, we obtain firm i's demand from consumers with strong expectations as

$$q^{\sigma}(p_i, p_j, \alpha_i^e, \lambda) =$$

$$\begin{cases} 1-\lambda & \text{if } \; p_j - p_i \geq b(1-2\alpha_i^e) + t \\ (1-\lambda)\left[\frac{1}{2} + \frac{b(2\alpha_i^e - 1)}{2t} + \frac{p_j - p_i}{2t}\right] & \text{if } \; b\,(1-2\alpha_i^e) - t < p_j - p_i < b(1-2\alpha_i^e) + t \\ 0 & \text{if } \; p_j - p_i \leq b\,(1-2\alpha_i^e) - t. \end{cases} \qquad (12)$$

Let us now turn to the consumers with weak expectations. Provided that prices are not too different, there exists a marginal consumer with address $x^{\omega}(p_A, p_B, \alpha_A^e, \lambda)$ who is indifferent between both firms' products. Straightforward calculations then give

$$x^{\omega}(p_A, p_B, \alpha_A^e, \lambda) = \frac{q^{\sigma}(p_A, p_B, \alpha_A^e)}{(\kappa - \lambda)} - \frac{(1-\kappa)}{2(\kappa - \lambda)} + \frac{p_B - p_A}{2b(\kappa - \lambda)}.$$

A consumer with weak expectations expects firm i's market share to be $q^{\sigma}(\cdot) + q^{\omega}(\cdot)$, where firm i's demand from consumers with weak expectations is $q^{\omega}(\cdot) = \lambda x^{\omega}(p_i, p_j, \alpha_i^e, \lambda)$ (whenever firms' prices are not too different). To derive the demand from consumers with weak expectations we have to consider the different price intervals as stated in (12).

We start with the first interval of (12), which requires $p_j - p_i \geq b(1 - 2\alpha_i^e) + t$, such that $q^{\sigma}(\cdot) = 1 - \lambda$. Given $q^{\sigma}(\cdot) = 1 - \lambda$, it holds that $q^{\omega}(\cdot) = \lambda$, if $2b(1-\lambda) + 2b(\lambda - \kappa) + b(\kappa - 1) + p_j - p_i \geq 0$, which implies $p_j - p_i \geq b(\kappa - 1)$. Note that for any α_i^e it holds that $p_j + b(1-\kappa) \geq p_j + b(2\alpha_i^e - 1 - \kappa)$. Hence, if $p_j - p_i \geq b(1 - 2\alpha_i^e) + t$, then $q^{\omega}(\cdot) = \lambda$. We now show that the demand from consumers with weak expectations cannot take other values. It holds that $q^{\omega}(\cdot) = 0$ if $2b(1-\lambda) + b(\kappa - 1) + p_j - p_i \leq 0$, which implies $p_j - p_i \leq -2b(1-\lambda) + b(1-\kappa)$. The comparison of $p_j + 2b(1-\lambda) + b(\kappa-1)$ and $p_j + b(2\alpha_i^e - 1 - \kappa)$ leads to the comparison of $\kappa - \lambda$ and $\alpha_i^e - 1$. If $\kappa > \lambda$, then $\kappa - \lambda > \alpha_i^e - 1$, hence, $p_j + 2b(1-\lambda) + b(\kappa - 1) > p_j + b(2\alpha_i^e - 1 - \kappa)$, which implies that $q^{\omega}(\cdot) = 0$ is not possible. We finally analyze whether $q^{\omega}(\cdot)$ can take any values on the interval $(0, \lambda)$. If $\kappa > \lambda$, then $0 < q^{\omega}(\cdot) < \lambda$ provided that $b(1-\kappa) + 2b(\lambda - 1) < p_j - p_i < b(\kappa - 1)$. Under the condition $\kappa > \lambda$ it holds that $p_j + 2b(1-\lambda) + b(\kappa - 1) > p_j + b(2\alpha_i^e - 1 - \kappa)$, which implies that $0 < q^{\omega}(\cdot) < \lambda$ is not possible.

We next consider the third interval of (12), where $p_j - p_i \leq b\,(1 - 2\alpha_i^e) - t$ with $q^{\sigma}(\cdot) = 0$. Which values can $q^{\omega}(\cdot)$ take on this interval? Note if $q^{\sigma}(\cdot) = 0$, then $q^{\omega}(\cdot) = 0$ follows, if $p_j - p_i \leq b(1-\kappa)$. For any α_i^e it holds that $p_j - b(1-\kappa) \leq p_j - b\,(1 - 2\alpha_i^e) + t$, which implies that $q^{\omega}(\cdot) = 0$. We now show that the demand from consumers with weak expectations cannot take any other value. If $q^{\sigma}(\cdot) = 0$, then $q^{\omega}(\cdot) = \lambda$ follows, if $p_j - p_i \geq 2b(\kappa - \lambda) + b(1-\kappa)$. The comparison yields that if $\kappa > \lambda$, then $p_j - [2b(\kappa - \lambda) + b(1-\kappa)] < p_j - [b\,(1 - 2\alpha_i^e) - t]$, which implies that $q^{\omega}(\cdot) = \lambda$ is not possible. There are also no values of $q^{\omega}(\cdot)$ on the interval $(0, \lambda)$. Provided that $\kappa > \lambda$, this is only possible if prices satisfy $b(1-\kappa) < p_j - p_i < b(1-\kappa) + 2b(\kappa - \lambda)$. As $b\,(1 - 2\alpha_i^e) - t < b(1-\kappa)$, $q^{\omega}(\cdot)$ cannot take any values on the interval $(0, \lambda)$.

We finally consider the intermediate interval of (12), where $b\,(1 - 2\alpha_i^e) - t < p_j - p_i <$

$b(1-2\alpha_i^e)+t$ holds, such that $q^\sigma(\cdot) \in (0, 1-\lambda)$. What values can $q^\omega(\cdot)$ take on this interval? As $\kappa > \lambda$, it holds that $q^\omega(\cdot) = \lambda$ if

$$p_j - p_i \geq \frac{b\kappa(\kappa-\lambda)}{1+\kappa-\lambda} - \frac{b\,(1-\lambda)\,(2\alpha_i^e-1)}{1+\kappa-\lambda},$$

while $q^\omega(\cdot) = 0$ if

$$p_j - p_i \leq -\frac{b\kappa(\kappa-\lambda)}{1+\kappa-\lambda} - \frac{b\,(1-\lambda)\,(2\alpha_i^e-1)}{1+\kappa-\lambda},$$

and, finally, $0 < q^\omega(\cdot) < \lambda$ when

$$-\frac{b\kappa(\kappa-\lambda)}{1+\kappa-\lambda} - \frac{b\,(1-\lambda)\,(2\alpha_i^e-1)}{1+\kappa-\lambda)} < p_j - p_i < \frac{b\kappa(\kappa-\lambda)}{1+\kappa-\lambda} - \frac{b\,(1-\lambda)\,(2\alpha_i^e-1)}{1+\kappa-\lambda}.$$

The comparison of $b\,(1-2\alpha_i^e) - t$ and $-b\kappa(\kappa-\lambda)/\,(1+\kappa-\lambda) - b\,(1-\lambda)\,(2\alpha_i^e-1)/(1+\kappa-\lambda)$ yields

$$\begin{aligned} & b\,(1-2\alpha_i^e) - t + \frac{b\kappa(\kappa-\lambda)}{1+\kappa-\lambda} + \frac{b\,(1-\lambda)\,(2\alpha_i^e-1)}{1+\kappa-\lambda} \\ = \quad & -\frac{2b\kappa\alpha_i^e}{1+\kappa-\lambda} \leq 0, \end{aligned}$$

with equality holding for $\alpha_i^e = 0$. The comparison of $b(1-2\alpha_i^e)+t$ and $-b\kappa(\kappa-\lambda)/\,(1+\kappa-\lambda) - b\,(1-\lambda)\,(2\alpha_i^e-1)/(1+\kappa-\lambda)$ yields

$$\begin{aligned} & b(1-2\alpha_i^e) + t + \frac{b\kappa(\kappa-\lambda)}{1+\kappa-\lambda} + \frac{b\,(1-\lambda)\,(2\alpha_i^e-1)}{1+\kappa-\lambda} \\ = \quad & \frac{2b\kappa\left[(1-\alpha_i^e) + (\kappa-\lambda)\right]}{1+\kappa-\lambda} > 0. \end{aligned}$$

In a similar way we get that $b\,(1-2\alpha_i^e) - t < b\kappa(\kappa-\lambda)/\,(1+\kappa-\lambda) - b\,(1-\lambda)\,(2\alpha_i^e-1)/(1+\kappa-\lambda)$ and $b\kappa(\kappa-\lambda)/\,(1+\kappa-\lambda) - b\,(1-\lambda)\,(2\alpha_i^e-1)/(1+\kappa-\lambda) \leq b(1-2\alpha_i^e)+t$, with equality holding for $\alpha_i^e = 1$. Hence, it follows that $0 \leq q^\omega(\cdot) \leq \lambda$.

Taking those results together, we can state firm i's demand from consumers with both weak and strong expectations as

$$q(p_i, p_j, \alpha_i^e, \lambda) = q^\sigma(p_i, p_j, \alpha_i^e, \lambda) + q^\omega(p_i, p_j, \alpha_i^e, \lambda) =$$

$$\begin{cases} 1 & \text{if} \quad \Delta p \geq b(1-2\alpha_i^e)+t \\ \frac{(1-\lambda)t + b(1-\lambda)(2\alpha_i^e-1) + (1-\lambda)(p_j-p_i) + 2\lambda t}{2t} & \text{if} \quad \frac{b\kappa(\kappa-\lambda) - b(1-\lambda)(2\alpha_i^e-1)}{1+\kappa-\lambda} < \Delta p < b(1-2\alpha_i^e)+t \\ \frac{\left[(1-\lambda)2\alpha_i^e + \kappa - 1\right]b + (p_j-p_i)}{2b(\kappa-\lambda)} & \text{if} \quad \frac{-b\kappa(\kappa-\lambda) - b(1-\lambda)(2\alpha_i^e-1)}{1+\kappa-\lambda} \leq \Delta p \leq \frac{b\kappa(\kappa-\lambda) - b(1-\lambda)(2\alpha_i^e-1)}{1+\kappa-\lambda} \\ \frac{(1-\lambda)t + b(1-\lambda)(2\alpha_i^e-1) + (1-\lambda)(p_j-p_i)}{2t} & \text{if} \quad b\,(1-2\alpha_i^e) - t < \Delta p < \frac{-b\kappa(\kappa-\lambda) - b(1-\lambda)(2\alpha_i^e-1)}{1+\kappa-\lambda} \\ 0 & \text{if} \quad \Delta p \leq b\,(1-2\alpha_i^e) - t, \end{cases}$$

where $\Delta p := p_j - p_i$.

We now turn to the equilibrium analysis. Inspecting the intervals of the market demand, we observe that the second interval and the fourth interval are symmetric as well as the first interval and the fifth interval. We can, therefore, confine our analysis to the first three intervals.

We start with the first interval where prices fulfill $p_j - p_i \geq b(1-2\alpha_i^e)+t$. In this interval firm i gets all the consumers (both strong-type and weak-type consumers). Hence, for expectations to be fulfilled it must hold that $\alpha_i^e = 1$. Note, if $\alpha_i^e = 1$, then

$$b(1-2\alpha_i^e)+t = \frac{b\kappa(\kappa-\lambda)}{1+\kappa-\lambda} - \frac{b\,(1-\lambda)\,(2\alpha_i^e-1)}{1+\kappa-\lambda}$$

follows, so that firm i's demand takes the form

$$q(p_i,p_j,1,\lambda) = q^{\sigma}(p_i,p_j,1,\lambda) + q^{\omega}(p_i,p_j,1,\lambda) =$$

$$\begin{cases} 1 & \text{if} \quad p_j - p_i \geq -b+t \\ \frac{(1-\lambda)2+\kappa-1}{2(\kappa-\lambda)} + \frac{(p_j-p_i)}{2b(\kappa-\lambda)} & \text{if} \quad -\frac{b\kappa(\kappa-\lambda)}{1+\kappa-\lambda} - \frac{b(1-\lambda)}{1+\kappa-\lambda} < p_j - p_i < -b+t \\ (1-\lambda)\left[\frac{1}{2} + \frac{b}{2t} + \frac{p_j-p_i}{2t}\right] & \text{if} \quad -b-t < p_j - p_i < -\frac{b\kappa(\kappa-\lambda)}{1+\kappa-\lambda} - \frac{b(1-\lambda)}{1+\kappa-\lambda} \\ 0 & \text{if} \quad p_j - p_i \leq -b-t. \end{cases}$$

By decreasing its price firm i cannot further increase its demand. Hence, in equilibrium it must hold that $p_j^* - p_i^* = -b+t$. Moreover, it must hold that $p_j^* = 0$. Otherwise, firm j could increase its profit by decreasing its price. Firm i must not have an incentive to increase its price, which implies that

$$\left.\frac{\partial \pi_i(p_i,p_j,1,\lambda)}{\partial p_i}\right|_{p_i=b(1-\kappa),\ p_j=0} = \frac{3\kappa-2\lambda-1}{2(\kappa-\lambda)} \leq 0$$

must hold. This yields the restriction $\kappa \leq (1+2\lambda)/3$. Hence, if $\lambda < \kappa \leq (1+2\lambda)/3$, a monopoly equilibrium exists, where the monopolist i sets the price $p_i^* = b(1-\kappa)$, while the competitor cannot do better than setting its price to zero. We, therefore, obtain the same monopoly equilibria as stated in part ii) of Proposition 3.

We next consider the second interval of the demand function, where prices must fulfill

$$[b\kappa(\kappa-\lambda) - b\,(1-\lambda)\,(2\alpha_i^e-1)]\,/\,(1+\kappa-\lambda) < p_j - p_i < b(1-2\alpha_i^e)+t. \tag{13}$$

We claim that there exists no equilibrium in this interval. We proceed by contradiction. If there exists an equilibrium, then each firm maximizes its profits which implies

$$p_i(\alpha_i^e) = \frac{t(3+\lambda)}{3(1-\lambda)} - \frac{b(1-2\alpha_i^e)}{3}$$

for firm i and

$$p_j(\alpha_i^e) = \frac{t(3-\lambda)}{3(1-\lambda)} + \frac{b(1-2\alpha_i^e)}{3}$$

for firm j. These prices determine the market share of firm i which becomes

$$q(p_i(\alpha_i^e), p_j(\alpha_i^e), \alpha_i^e, \lambda) = \frac{(3+\lambda)\,t}{(1-\lambda)6t} + \frac{(1-\lambda)b(2\alpha_i^e-1)}{6t}.$$

In equilibrium it must hold that $q(p_i(\alpha_i^e), p_j(\alpha_i^e), \alpha_i^e, \lambda) = \alpha_i^e$, which yields the market share of firm i

$$\widetilde{\alpha}_i = \frac{(3+\lambda)\kappa - 1 + \lambda}{2(3\kappa+\lambda-1)}. \tag{14}$$

Using this market share we can compute the difference in firms' prices as

$$p_j(\widetilde{\alpha}_i) - p_i(\widetilde{\alpha}_i) = -\frac{2t\lambda\kappa}{(1-\lambda)(3\kappa+\lambda-1)}. \tag{15}$$

We now show that the difference $p_j(\widetilde{\alpha}_i) - p_i(\widetilde{\alpha}_i)$ never lies within the assumed price interval. Substituting (14) into the right-hand side of the second inequality of (13) we obtain

$$b(1-2\widetilde{\alpha}_i) + t = -\frac{b\kappa(1-3\kappa)}{3\kappa+\lambda-1}.$$

We can then rewrite the second inequality of (13) as

$$\frac{t\,[\kappa\lambda - (3\kappa+\lambda-1)]}{(1-\lambda)(3\kappa+\lambda-1)} < 0,$$

which implies that $\kappa\lambda - (3\kappa+\lambda-1) < 0$ must hold. Substituting (14) into the left-hand side of the first inequality of (13), we obtain after some calculations the condition

$$\frac{t\,[\kappa\lambda - (3\kappa+\lambda-1)]}{(1-\lambda)(1+\kappa-\lambda)(3\kappa+\lambda-1)} > 0,$$

which implies that $\kappa\lambda-(3\kappa+\lambda-1) > 0$ must hold. Note that the conditions $\kappa\lambda-(3\kappa+\lambda-1) < 0$ and $\kappa\lambda-(3\kappa+\lambda-1) > 0$ cannot hold simultaneously. Hence, an equilibrium with prices p_j and p_i that fulfill (13) does not exist.

We, finally, turn to the third interval of the demand function which requires

$$\frac{-b\kappa(\kappa-\lambda) - b\,(1-\lambda)\,(2\alpha_i^e-1)}{(1+\kappa-\lambda)} \leq p_j - p_i \leq \frac{b\kappa(\kappa-\lambda) - b\,(1-\lambda)\,(2\alpha_i^e-1)}{(1+\kappa-\lambda)}. \tag{16}$$

Maximization of firm i's profit yields

$$p_i(p_j, \alpha_i^e) = \frac{p_j + b\,[(1-\lambda)2\alpha_i^e + \kappa - 1]}{2} \tag{17}$$

and maximization of firm j's profit yields

$$p_j(p_i, \alpha_i^e) = \frac{p_i + b\,[2(1-\lambda) - 2(1-\lambda)\alpha_i^e + \kappa - 1]}{2}. \tag{18}$$

Combining both (17) and (18), we can write firms' prices as a function of consumer expectations as

$$p_i(\alpha_i^e) = \frac{b\,[3\kappa - 2\lambda - 1 + 2(1-\lambda)\alpha_i^e]}{3}, \tag{19}$$

$$p_j(\alpha_i^e) = \frac{b\,[3\kappa - 4\lambda + 1 - 2(1-\lambda)\alpha_i^e]}{3}, \tag{20}$$

which yield firm i's market share

$$q(p_i(\alpha_i^e), p_j(\alpha_i^e), \alpha_i^e, \lambda) = \frac{2(1-\lambda)\alpha_i^e + 3\kappa - 2\lambda - 1}{6(\kappa - \lambda)}.$$

Equating $q(p_i(\alpha_i^e), p_j(\alpha_i^e), \alpha_i^e, \lambda) = \alpha_i^e$, we get $\alpha_i^* = 1/2$. Substituting $\alpha_i^* = 1/2$ into (19) and (20) yields the equilibrium prices as stated in Proposition 3; namely $p_A^{d,ME} = p_B^{d,ME} = b(\kappa - \lambda)$. Note finally, that these prices together with $\alpha_i^* = 1/2$ satisfy the two inequalities of (16). *Q.E.D.*

References

d'Aspremont, C., Gabszewicz, J.-F., and Thisse, J.-F. (1979), On Hotelling's "Stability in Competition," Econometrica 47, 1145-1150.

Baake, P. and Boom, A. (2001), Vertical Product Differentiation, Network Externalities, and Compatibility Decisions, International Journal of Industrial Organization 19, 267-284.

Becker, G. (1991), A Note on Restaurant Pricing and Other Examples of Social Influences on Prices, Journal of Political Economy 99, 1109-1116.

Corneo, G. and Jeanne, O. (1997a), Conspicuous Consumption, Snobbism and Conformism, Journal of Public Economics 66, 55-71.

Corneo, G. and Jeanne, O. (1997b), Snobs, Bandwagons, and the Origin of Social Customs in Consumer Behavior, Journal of Economic Behavior and Organization 32, 333-347.

David, P.A. and Greenstein, S. (1990), The Economics of Compatibility Standards: An Introduction to Recent Research, Economics of Innovation and New Technologies 1, 3-41.

Farrell, J. and Katz, M.L. (1998), The Effects of Antitrust and Intellectual Property Law on Compatibility and Innovation, Antitrust Bulletin 43, 609-650.

Farrell, J. and Klemperer, P. (2007), Coordination and Lock-in: Competition with Switching Costs and Network Effects, pp. 1967-2072, in: M. Armstrong and R. Porter (eds.), Handbook of Industrial Organization, Vol. 3, Elsevier, Amsterdam.

Farrell, J. and Saloner, G. (1992), Converters, Compatibility, and the Control of Interfaces, Journal of Industrial Economics 40, 9-35.

Glazer, A., Kanniainen, V., and Mustonen, M. (2006), When a Loser Gains: Free Riding in the Innovation of Network Goods, Journal of Economics 87, 55-71.

Grilo, I., Shy, O., and Thisse, J.-F. (2001), Price Competition when Consumer Behavior is Characterized by Conformity or Vanity, Journal of Public Economics 80, 385-408.

Katz, M.L. and Shapiro, C. (1985), Network Externalities, Competition, and Compatibility, American Economic Review 75, 424-440.

Kristiansen, E. and Thum, M. (1997), R&D Incentives in Compatible Networks, Journal of Economics 65, 55-78.

Shapiro, C. and Varian, H. (1999), Information Rules, Harvard Business School Press, Boston, Massachussets.

Shy, O. (2001), The Economics of Network Industries, Cambridge University Press, Cambridge, U.K.

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