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Price-Dependent Demand in Spatial Models*

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Abstract

This paper introduces price-dependent individual demand into the circular city model of product differentiation. We show that for any finite number of firms, a unique symmetric price equilibrium exists provided that demand functions are not "too" convex. As in the case of unit demand, the number of firms under free entry decreases in the fixed cost of entry while increases in the transportation cost of consumers. However, this number is no longer always in excess of the socially optimal level. Insufficient entry occurs when the fixed and transportation costs are high.

Keywords: Spatial Models; Price-Dependent Demand; Horizontal Product Differentiation; Demand Elasticity; Excess Entry Theorem.

JEL-Classification: L11, L13, R1.

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1 Introduction

Spatial models of product differentiation have been a workhorse in Industrial Organization and Regional Science. Typically, the linear city model (Hotelling, 1929) has been used to study location decisions by firms while the circular city model (Vickrey, 1964 and Salop, 1979) has been used to study entry decisions and market structure. Given such importance and popularity, almost all aspects of spatial models have been thoroughly studied and different variants have been proposed with changes in the distribution of consumers and in the structures of production costs and transportation costs. See, e.g., Anderson et al. (1992) for a comprehensive treatment.[1] However, the original, and quite restrictive assumption on individual consumer demand has been mostly maintained. Namely, each consumer only demands a single unit of a differentiated product provided that it is offered below reservation price. For many products such a unit demand schedule is inadequate. A special case of price dependent demand (constant elasticity of demand) has been studied in Gu and Wenzel (2009).[2] It is the aim of the present paper to incorporate *general* price-dependent individual demand into the circular city model, and to investigate the consequences of this modification on the validity of previous results.

Our main contribution is two-fold. Firstly, we show that provided that individual demand functions are not "too" convex, bringing in price-dependent demand does not damage the existence of price equilibrium. Indeed, with this mild restriction on the demand function, there exists a unique symmetric price equilibrium for any finite number of firms. This may come as a surprise as the discontinuity in a firm's profit function produced by leapfrogging prices may make best response correspondences ill behaved. However, with the help of a constructed constant-elasticity demand function, we found such leapfrogging is impossible at candidate equilibrium prices.

Secondly, we characterize the model outcome and welfare properties. For a given demand function, the following results of the standard circular city model are confirmed: In the pricing stage, equilibrium price and firm profits are decreasing in the number of firms while increasing in the transportation cost. Under free entry, the number of firms decreases, while equilibrium price and industry total revenue increase in the fixed cost

[1] In particular, since d'Aspremont et al. (1979), the structure of transportation costs has received the most attention. Considerations of consumer distribution and production costs can be found in Shilony (1981), Neven (1986), Calvó-Armengol and Zenou (2002) and Matsumura and Okamura (2006b) among others. The assumption of single product firms is relaxed in, e.g., Janssen et al. (2005).

[2] Below we will discuss the relationship between this paper and our previous work in detail.

of entry; all of them are increasing in the transportation cost. Our welfare results under free entry, however, are very different. The well known excess entry theorem of circular city models states that in a free-entry equilibrium, there are always more firms entering into the market than would be desirable from a welfare point of view.[3] That is, there is excessive entry into the market irrespective of other parameters in the model. As firms are usually assumed to be single product firms, this outcome can also be interpreted as an excess of product variety provided in the market.[4] However, entry in the current model can be insufficient, optimal, and excessive depending on entry and transportation costs. In particular, for any demand function considered in this paper, and for any given fixed cost, market entry is insufficient when the transportation cost is high enough. For a given transportation cost, insufficient entry may also result if the fixed cost is high.

Therefore, our paper not only provides a modeling framework for including general price-dependent individual demand into standard models in an analytically tractable manner, but also shows that such a generalization retains the essence of spatial models as the comparative statics results suggest. The significance of our approach is evident in two perspectives. First, individual demand of many differentiated products are price-dependent, such as, confections, alcoholic beverages.[5] Normally, for these products consumers also have their favorite brands. For these markets, spatial models are well suited and a researcher need not choose, for example, the representative consumer model over a spatial one just because demand is price-dependent.[6] Second, using a circular city model does not automatically mean that the number of variety provided in a free market is excessive any more. True policy implications can be derived by inspecting entry costs and transportation costs using the current framework.

The intuition behind our welfare results is the following. When setting a price for the product a firm has to take two effects into account. A decrease in price increases its market share as well as the quantity sold to each of its customers. This second effect—

[3]In his original contribution Salop (1979) did not stress this result, but rather states that "this result of too many brands is not robust, but rather depends crucially on the distribution of consumers and preferences". Though, he does not show it formally.

[4]With respect to variants of the standard circular city model, Matsumura and Okamura (2006b) find this excess entry result holds for a broad class of transport and production cost functions. Matsumura (2000) shows that there are cases when the excess entry result does not hold if the integer problem is considered. When consumers are not uniformly distributed, Calvó-Armengol and Zenou (2002) find entry can be insufficient.

[5]On the other hand, a unit-demand schedule is appropriate for products such as household appliances, automobiles, etc.

[6]The demand of a representative consumer is price-dependent but market competition is global. See Spence (1976), Dixit and Stiglitz (1977) and Anderson and de Palma (2000).

not present in the standard circular city model with unit demand—makes firms more aggressive in price competition, and hence leads to a lower equilibrium price than in the standard model. This, in turn, leads to lower profits and reduces the incentives to enter the market. Thus, considering price-dependent demand leads to a downward correction of the number of firms which are active in the market and therefore, free entry can be insufficient. Especially, when entry and transportation costs are high, market entry falls short of the socially optimal level.

Several recent papers have aimed at introducing price-dependent demand into spatial models and analyzing the consequences but we are not aware of a paper that systematically addresses this aspect and provides a thorough analysis. In our previous work, Gu and Wenzel (2009), we relaxed the unit demand schedule by letting individual demand exhibit a *constant price elasticity*. Nevertheless, that generalization still lays strong restrictions on the underlying consumer preference. The assumption that elasticity is independent of the price also makes other exogenous variables in the model play no role in welfare ranking. That is, for a given constant-elasticity demand function, whether entry is insufficient or excessive solely depends on this elasticity, a variable that is not present in standard models, and hence, no understanding of entry and transportation costs' impacts on market efficiency was gained.[7] In comparison, while Gu and Wenzel (2009) demonstrate insufficient entry is possible under a class of special demand functions, the current paper explains why and when entry is insufficient with references to existing parameters. Equally important is that we additionally show the existence of price equilibrium under general price-dependent demand.

Boeckem (1994) and Rath and Zhao (2001) depart from completely inelastic demand in the Hotelling setup. Boeckem (1994) considers a setup where individual consumers demand one unit of a product but they differ in their reservation prices. Therefore, demand is price-dependent only from a firm's perspective. Rath and Zhao (2001) use a setup where each consumer has a linear demand function. Both papers show that the principle of maximum differentiation may not hold if considering price-dependent demand. Anderson and de Palma (2000) propose a model that combines features of localized competition and representative consumer models where competition is global. In this model, individual demand is price-dependent with a constant price elasticity.

[7] As can be seen in Proposition 2 in Section 5.1, these two cost variables affect the welfare ranking of free entry and the first-best benchmark insofar as they affect the equilibrium price elasticity of demand. Clearly, in the very special case of constant elasticity, these two cost variables play no role in welfare ranking. On the other hand, in the current, more general setting, equilibrium price elasticity of demand depends on equilibrium price which in turn depends on the two cost variables.

It should be noted that Matsumura and Okamura (2006a) introduce a linear demand function into a circular city model with delivered-price competition, and also find entry can be insufficient when the fixed cost is large.[8] In contrast to the above mentioned earlier contributions, the present paper does not rely on specific functional forms of consumer demand and considers the circular city structure with firms competing in mill prices.

The remainder of the paper is structured as follows. Section 2 outlines the model, introduces our main assumption and discusses the model setup. Section 3 analyzes price competition and establishes the existence and uniqueness of a symmetric price equilibrium. The impacts of the number of firms and the transportation cost on this equilibrium price are discussed. In Section 4, we study the number of firms that enter in a free-entry equilibrium and derive comparative statics results. Section 5 compares this free-entry equilibrium both with the first-best and with the second-best welfare benchmark. Section 6 collects a few concluding remarks.

2 The model

2.1 Model setup

There is a unit mass of consumers who are uniformly located on a circle with circumference one. The location of a consumer is denoted by x. Consumers have to incur costs of mismatch (transportation costs) if the product's attributes do not match consumers' preferences; these costs are linear in distance[9] with a marginal rate of $t > 0$, and do not depend on the quantity consumed.[10]

Our modification of the standard circular city model (as outlined in, e.g., Tirole, 1988) is in individual consumer demand. Given that a consumer has decided to buy the product at a price p, he buys a quantity $q(p)$ of that product. The individual demand

[8] In the Appendix to a study of product variety under different pricing regimes and spatial contestabilities, Norman and Thisse (1996) also consider a circular city model with linear demand. The authors show numerically that insufficient entry occurs when relocation is prohibitively costly and the fixed cost is high.

[9] Our main welfare results hold also under quadratic transportation costs. Calculations are available from the authors upon request.

[10] Transportation costs are one time costs independent of the quantity. As an interpretation these could be costs for driving to a shopping mall. Alternatively, one could also assume transportation costs to depend on the quantity. This would be a plausible assumption if the horizontal dimension is interpreted as a taste dimension.

function $q(p) : [0, \hat{p}] \to [0, \hat{q}]$ is continuous and differentiable on $[0, \hat{p}]$ with $q' < 0$, where $0 < \hat{p} < \infty$ is the price at which demand becomes zero, and $0 < \hat{q} < \infty$ is the maximum demand at zero price.[11] $q(p)$ is identical across firms and consumers. A consumer located at x purchasing from a firm located at x_i at a price $p_i < \hat{p}$ obtains a surplus

$$V + \int_{p_i}^{\hat{p}} q(s)\, ds - t|x - x_i|, \tag{1}$$

where V is the gross utility from consuming the differentiated product. We assume that V is large enough so that the market is covered.[12]

The differentiated product is offered by an oligopolistic industry with $n \geq 2$ firms ($i = 1, 2, ..., n$) each producing a single variant at a constant marginal cost (which is normalized to 0). The firms are located equidistantly on the unit circle, that is, the distance between any two neighboring firms is $\frac{1}{n}$. To model competition in this market, we study the following three stage game and solve it backwards. In the first stage, firms may enter the market by incurring a fixed cost $f > 0$. In the second stage, firms compete in prices. In the third stage, consumers choose a supplier of the differentiated product and the quantity.

2.2 Main assumption

Let the absolute value of price elasticity of demand be

$$\varepsilon(p) := -\frac{pq'(p)}{q(p)}. \tag{2}$$

Differentiability of $q(p)$ implies $\varepsilon(p)$ is continuous, $\varepsilon(0) = 0$, and $\varepsilon(p) \to \infty$ as $p \to \hat{p}^-$. Our main assumption which is assumed throughout the paper is stated in terms of $\varepsilon(p)$.[13]

[11] In this paper, we use both Newton's and Leibniz' notation of differentiation. In particular, Newton's notation is applied to operations w.r.t. p while Leibniz' is reserved for stating various comparative statics results.

[12] The assumption that the market is covered has consequences on the analysis of price competition for a given number of firms. Under free entry, however, this assumption is not very restrictive.

[13] This assumption appears commonly in the management literature to ensure that the revenue function $R(p)$ is strictly unimodal. See, e.g., Ziya et al. (2004) and the references therein. In particular, Ziya et al. (2004) provide a comparison of Assumption 1 to other popular assumptions, and find in the region such that $p \in (0, p^m]$ it is weaker than the revenue function being strictly concave in *demand*.

Assumption 1 *The absolute value of price elasticity of demand* $\varepsilon(p)$ *is strictly increasing in* $p \in [0, \hat{p})$.

Let $R(p) := pq(p)$ be the revenue function associated with $q(p)$. The following lemma collects a few immediate consequences of this assumption that are of the most interest to us.

Lemma 1 *There exists a unique price* $p^m \in (0, \hat{p})$ *that maximizes* $R(p)$ *on* $[0, \hat{p}]$. *Moreover,* $R(p)$ *is strictly increasing in* $[0, p^m)$, *and* $\varepsilon(p^m) = 1$.

Proof. First, $R'(p) = q(p) + pq'(p) = (1 - \varepsilon(p))\, q(p)$. Since $\varepsilon(p)$ is continuous and strictly increasing from 0 to ∞, there exists a unique $p^m \in (0, \hat{p})$ such that $\varepsilon(p^m) = 1$ and hence $R'(p^m) = 0$. Furthermore, $R(0) = R(\hat{p}) = 0$, and $R'(p) > \{<$, resp.$\}\ 0$ for p in $(0, p^m)$ $\{(p^m, \hat{p})$, resp.$\}$. Therefore, $R(p)$ is strictly quasi-concave and the lemma follows. ■

2.3 Discussion of model setup

With respect to our model setup, several remarks are in order. First, as far as modeling is concerned, it is straightforward to allow for demand functions with a constant price elasticity $0 < \varepsilon < 1$ for p in $(0, p^m)$ as in Gu and Wenzel (2009) while keeping the existence and uniqueness of a symmetric price equilibrium intact.[14] But as discussed in the Introduction, the efficiency of market entry in this case is independent of entry and transportation costs. Indeed, for the welfare results in Section 5 to apply, it is important that $\varepsilon(p)$ has enough variation as p changes.

Second, from Section 3.4 onwards, we assume that $q(p)$ is twice continuously differentiable so that $\varepsilon(p)$ is differentiable on $[0, \hat{p})$. This assumption will greatly facilitate the exposition of comparative statics and welfare results. In particular, Assumption 1 can be efficiently written as $\varepsilon'(p) > 0$. Nevertheless, for the existence of a unique symmetric price equilibrium in the second stage, this is not needed.[15]

Finally, and importantly, we note that when $q(p)$ is twice continuously differentiable, Assumption 1 can be interpreted as demand functions not being "too" convex. For $p \in$

[14]For a given ε, to ensure existence, p^m should be larger than $\left[\frac{t}{n}(1-\varepsilon)\right]^{\frac{1}{1-\varepsilon}}$.

[15]To show existence, we only require payoff functions to be quasiconcave in own prices, for which differentiability of the demand function is already sufficient. See (19) in Appendix A.1.

$(0, \hat{p})$, it is equivalent to

$$q'' < \frac{(q')^2}{q} + \frac{(-q')}{p}. \tag{3}$$

As the right-hand side is strictly positive, this restriction has no bite on concave demand functions. We note that any demand function of the form

$$q(p) = a - bp^{\gamma}, \tag{4}$$

where $\gamma > 0$ and $a, b > 0$, satisfies this assumption.[16] This class includes the widely used linear ($\gamma = 1$) and quadratic ($\gamma = 2$) demand functions. Moreover, for $\gamma \in (0,1)$, $a - bp^{\gamma}$ is convex. Condition (3) is also weaker than some other "not-too-convex" assumptions, such as, $q(p)$ being log-concave; in the price interval where $\varepsilon(p) \leq 1$, it is weaker than ρ-concavity for $\rho > -1$.[17]

3 Price equilibrium

In this section we first analyze consumer choice in the third stage, and then study price competition in the second stage for a given, finite number ($n \geq 2$) of firms.

3.1 Marginal consumer and demand

Given the symmetric structure of the model, we seek for a symmetric equilibrium. Therefore, we derive demand of a representative firm i which for convenience is designated to be located at zero. Suppose that this firm charges a price of p_i while all remaining firms charge a price of p_o. Then the marginal consumer is the one who is indifferent between buying from firm i and its neighboring firm located at $\frac{1}{n}$. Using (1) the marginal consumer ($\bar{x}$) is given implicitly by

$$\int_{p_i}^{\hat{p}} q(s)\, ds - t\bar{x} = \int_{p_o}^{\hat{p}} q(s)\, ds - t\left(\frac{1}{n} - \bar{x}\right),$$

[16] In this case, $\varepsilon'(p) = \frac{ab\gamma^2 p^{\gamma-1}}{(a-bp^{\gamma})^2}$ which is clearly strictly positive for all $p \in \left(0, \left(\frac{a}{b}\right)^{\frac{1}{\gamma}}\right)$.

[17] Roughly, a demand function $q(p)$ is said to be ρ-concave if q^{ρ} (resp. $-q^{\rho}$) is concave for $\rho > 0$ (resp. $\rho < 0$). The case of $\rho = 0$ corresponds to log-concavity. See Caplin and Nalebuff (1991). Given differentiability, ρ-concavity means $q'' \leq (1-\rho)(q')^2/q$. For $\rho \geq 0$, it is clear that ρ-concavity implies (3) for all $p \in (0, \hat{p})$. For $-1 < \rho < 0$ *and* $\varepsilon(p) \leq 1$, $q'' \leq (1-\rho)(q')^2/q = (q')^2/q + (-\rho\varepsilon)(-q'/p) < (q')^2/q + (-q'/p)$. Therefore, ρ-concavity implies (3) in this case as well.

or explicitly by

$$\bar{x}(p_i) = \frac{1}{2n} + \frac{1}{2t}\int_{p_i}^{p_o} q(s)\,ds. \tag{5}$$

When p_i increases, $\bar{x}$ is moving closer to firm i, and hence its market share decreases.

As each firm faces two adjacent firms, the measure of consumers choosing to buy from firm i is $2\bar{x}$. According to the individual demand function, each consumer buys an amount of $q(p_i)$. Hence total demand at firm i is $D_i(p_i) = 2\bar{x}(p_i) \cdot q(p_i)$. In contrast to the standard model with completely inelastic demand, total demand now consists of two parts: market share and quantity per consumer. When choosing prices firms have to take into account both effects. An increase in price reduces market share as well as the quantity that can be sold to each customer. This second effect is not present in the standard model.

3.2 Price equilibrium

With zero production costs, the profit of the representative firm i is given by:

$$\Pi_i(p_i) = D_i(p_i) \cdot p_i = \left[\frac{1}{n} + \frac{1}{t}\int_{p_i}^{p_o} q(s)\,ds\right] q(p_i)\,p_i. \tag{6}$$

To find profit maximizing price p_i, we first derive the first order derivative,

$$\Pi_i'(p_i) = -\frac{p_i\left[q(p_i)\right]^2}{t} + \left[\frac{1}{n} + \frac{1}{t}\int_{p_i}^{p_o} q(s)\,ds\right]\left[q(p_i) + p_i q'(p_i)\right]. \tag{7}$$

By setting equation (7) to zero and using (2) we obtain the following necessary condition,

$$p_i q(p_i) = \left[\frac{t}{n} + \int_{p_i}^{p_o} q(s)\,ds\right]\left[1 - \varepsilon(p_i)\right].$$

For the moment let us suppose that a symmetric price equilibrium exists. Applying symmetry to the above first-order condition, a symmetric price equilibrium is characterized by:

$$R(p^*) = \frac{t}{n}\left[1 - \varepsilon(p^*)\right], \tag{8}$$

where p^* denotes the symmetric equilibrium price. We use this condition to state corresponding equilibrium profits. Inserting (8) into (6) we have

$$\Pi(p^*) = \frac{t}{n^2}[1 - \varepsilon(p^*)]. \tag{9}$$

It can be seen immediately that there is a negative relationship between equilibrium demand elasticity (or price) and equilibrium profit.

3.3 Equilibrium existence and uniqueness

Equilibrium existence and uniqueness are established in this part. We start with the easier task of uniqueness. Note that $R(p^*) \geq 0$ and $\varepsilon(p^m) = 1$, for (8) to hold, by Assumption 1, p^* can only be in $[0, p^m]$.

Lemma 2 *For any $2 \leq n < \infty$ and $t > 0$, there exists a unique p^* that satisfies (8). Furthermore, $p^* \in (0, p^m)$ and $\varepsilon(p^*) \in (0, 1)$.*

Proof. For ease of exposition, let

$$\Delta(p) := R(p) - \frac{t}{n}[1 - \varepsilon(p)]. \tag{10}$$

$\Delta(p)$ is continuous. Since $\varepsilon(0) = 0$, $2 \leq n < \infty$ and $t > 0$, $\Delta(0) = -\frac{t}{n} < 0$. By $\varepsilon(p^m) = 1$, $\Delta(p^m) = R(p^m) > 0$. Moreover, by Lemma 1 and Assumption (1), $\Delta(p) = R(p) + \frac{t}{n}\varepsilon(p) - \frac{t}{n}$ is strictly increasing in $p \in [0, p^m]$. Therefore, there exists a unique $p^* \in (0, p^m)$ that satisfies $\Delta(p^*) = 0$. Since $1 - \varepsilon(p) \leq 0$ for $p \in (p^m, \hat{p}]$, $\Delta(p) > 0$ in this interval. Therefore, there is no other solution. It is straightforward to see $\varepsilon(p^*) \in (0, 1)$.
■

This result shows that there is one and only one candidate for a symmetric price equilibrium. Left to be verified is that the symmetric strategy profile $p_{i=1,2,\ldots,n} = p^*$ does qualify as a price equilibrium of the n-player game. In fact, this is the case, and hence (8) becomes a necessary and sufficient condition for a symmetric price equilibrium in the second stage.

Theorem 1 *For any given finite number of firms $n \geq 2$, there exists a unique symmetric price equilibrium in which the symmetric price is the unique solution to $R(p^*) = \frac{t}{n}[1 - \varepsilon(p^*)]$.*

Proof. See Appendix A.1. ■

That $p_{i=1,2,...,n} = p^*$ is a price equilibrium is shown in two steps. Fix a firm i and suppose all other firms are charging p^*. The first step is to show that when i' s two marginal consumers are located between its immediate neighbors and itself, i's profit (6) is maximized when $p_i = p^*$. Indeed, (6) is strictly quasiconcave in p_i. Firstly, the observation that a firm never charges a price above p^m allows us to focus on $p_i \in [0, p^m]$. Secondly, using the fact that $\Delta(p)$–as in (10)–is strictly increasing in $[0, p^m]$, (6) is found strictly increasing in $[0, p^*)$ and strictly decreasing in $(p^*, p^m]$. Therefore, p^* is the only turning point of the continuous function (6), and hence the unique best response of firm i in this case.[18]

The second step is to take care of the discontinuity, if any, in firm i's profit (or market share) when it leapfrogs its immediate neighbors. It, however, turns out impossible for firm i to leapfrog its immediate competitors, that is, i's marginal consumers will never be located outside the distance between itself and its immediate neighbors. This result is shown by constructing an auxiliary demand function with a constant elasticity under which leapfrogging is barely possible.[19] Given that our demand function is less elastic in lower prices, leapfrogging is shown impossible. The intuition of this impossibility is that the equilibrium price is already adjusted to the exogenous variables n and t, and is located in the inelastic segment of the demand function. Even if a firm offered zero price, traveling an additional distance of $\frac{1}{n}$ is not attractive to consumers. Since no generality is lost in picking firm i, these two steps establish that $p_{i=1,2,...,n} = p^*$ is indeed a price equilibrium, and hence we proved equilibrium existence by identifying one. In light of Lemma 2, there is no other symmetric price equilibrium in the second stage. Details appear in Appendix A.1.

3.4 Properties of price equilibrium

Here we study the properties of the price equilibrium. Lemma 3 below states the effects of the number of firms which are active in the market and of the transportation cost on equilibrium price, equilibrium price elasticity, industry total profit and individual firm profits.

[18] Had $\varepsilon(p)$ been only weakly increasing, provided that $\varepsilon(p) < 1$for $p \in [0, p^m)$, $R(p)$ is strictly increasing, and hence $\Delta(p)$ would still be strictly increasing. Thus, weakly increasing $\varepsilon(p)$ suffices.

[19] Under this auxiliary demand function, the only possible leapfrogging price is zero at which the firm earns zero profit. Therefore, leapfrogging is never profitable even under demand functions with a weakly increase $\varepsilon(p)$.

Lemma 3 *Comparative statics in price equilibrium.*

1. *Equilibrium price p^*, price elasticity of demand $\varepsilon(p^*)$, industry total profit $R(p^*)$ and firm profits $\Pi(p^*)$* decrease *in the number of entrants n, that is, $\frac{\partial p^*}{\partial n} < 0$, $\frac{\partial \varepsilon(p^*)}{\partial n} < 0$, $\frac{\partial R(p^*)}{\partial n} < 0$ and $\frac{\partial \Pi(p^*)}{\partial n} < 0$.*

2. *Equilibrium price p^*, price elasticity of demand $\varepsilon(p^*)$, industry total profit $R(p^*)$ and firm profits $\Pi(p^*)$* increase *in the transportation cost t, that is, $\frac{\partial p^*}{\partial t} > 0$, $\frac{\partial \varepsilon(p^*)}{\partial t} > 0$, $\frac{\partial R(p^*)}{\partial t} > 0$ and $\frac{\partial \Pi(p^*)}{\partial t} > 0$.*

Proof. See Appendix A.2. ■

Not surprisingly, the larger the number of firms, the more competition, the lower the equilibrium price, and hence the lower the equilibrium price elasticity of demand. As the price is lower, the industry total profit decreases and individual profits decrease even faster. The impact of the transportation cost on equilibrium price and profits is also the same as in standard location models. They increase in the transportation cost. That t and n have opposite effects is also evident from (8) as they appear as a quotient.

4 Free Entry

Until now the analysis has treated the number of firms which offer differentiated products as exogenously given. In this section, we investigate the number of active firms when it is endogenously determined by a zero profit condition in the first stage of market entry. We assume that to enter, a firm has to incur an entry cost or fixed cost of $f > 0$. Additionally, we treat the number of entrants as a continuous variable. Setting equation (9) equal to f determines implicitly the number of firms that enter. We denote this number by n^c and the resulting equilibrium price $p^*_{n^c}$:

$$\frac{t}{(n^c)^2}\left[1 - \varepsilon\left(p^*_{n^c}\right)\right] = f. \tag{11}$$

In general, it is not possible to express the number of entrants explicitly as no specific demand function is assumed. Nevertheless, we can determine the signs of changes in the endogenous number of firms as the two exogenous variables change.

4.1 The impact of the fixed cost

Keep t constant. The effects of the fixed cost is easy to detect. Equation (11) is just $\Pi\left(p_{n^c}^*\right) = f$. As the fixed cost increases, an active firm's profit has to rise. From Lemma 3 we know that this is only possible if the number of firms decreases. Therefore, an increase in the fixed cost reduces entry: $\frac{\partial n^c}{\partial f} < 0$. As f does not appear in (8), the fixed cost impacts equilibrium price only through the number of firms. Hence, by Lemma 3, equilibrium price, elasticity, and of course, an individual firm's revenue are increasing in the fixed cost: $\frac{\partial p_{n^c}^*}{\partial f} > 0$, $\frac{\partial \varepsilon\left(p_{n^c}^*\right)}{\partial f} > 0$, $\frac{\partial \Pi\left(p_{n^c}^*\right)}{\partial f} = 1$. As a consequence of a rising equilibrium price, the industry is earning more revenue ($R\left(p_{n^c}^*\right)$). Therefore, although the number of firms is decreasing, the industry total fixed cost is increasing: $n^c f = R\left(p_{n^c}^*\right)$.

When the fixed cost rises, the number of entrants n^c will eventually reach 2. For a given transportation cost t, let $F\left(t\right) := \frac{t}{4}\left[1 - \varepsilon\left(p_{n=2}^*\right)\right] = \frac{R\left(p_{n=2}^*\right)}{2}$ be the fixed cost at which exactly two firms enter into the market. We note that $F\left(t\right)$ defined in this way depends on the demand function $q\left(p\right)$ which is a primitive of our model, and on t which is also implicitly included in the definition of $p_{n=2}^*$. For given $q\left(p\right)$ and t, by (8) we can find $p_{n=2}^*$ and thus $F\left(t\right)$. When $F\left(t\right) < f < R\left(p^m\right)$, only one firm can make a profit in the market while when $f > R\left(p^m\right)$, no firm would enter.[20] As we do not discuss integer problems, in the following we only consider the case $f \in \left(0, F\left(t\right)\right]$ in which there is a one-to-one relationship between f and $p_{n^c}^*$ because $\frac{\partial p_{n^c}^*}{\partial f} > 0$. However, as it will turn out to be important, the upper bound on $\varepsilon\left(p_{n^c}^*\right)$ is $\varepsilon\left(p_{n=2}^*\right)$. Therefore, as f decreases from $F\left(t\right)$, $\varepsilon\left(p_{n^c}^*\right)$ decreases from $\varepsilon\left(p_{n=2}^*\right) < 1$.

Another important observation is that $F\left(t\right)$ strictly increases in t. To see this, note that $R\left(p_{n=2}^*\right)$ is the industry revenue for a given number of firms, namely, 2. By Lemma 3, $R\left(p_{n=2}^*\right)$ is strictly increasing in t, and so is $F\left(t\right) = \frac{R\left(p_{n=2}^*\right)}{2}$. This means, when the transportation cost increases, higher levels of fixed cost can be considered.

4.2 The impact of the transportation cost

The implication of the transportation cost is less straightforward as it appears in both (8) and (11). However, since both functions hold in equilibrium, we can combine them

[20] Since $R\left(p_{n=2}^*\right) < R\left(p^m\right)$, $F\left(t\right) < \frac{R\left(p^m\right)}{2}$.

by substituting out n^c to get

$$R^2\left(p^*_{n^c}\right) = ft\left[1-\varepsilon\left(p^*_{n^c}\right)\right]. \tag{12}$$

We then deduce the impact of t on $p^*_{n^c}$ first. Namely, keep f constant and differentiate both sides of (12) with respect to t. We have

$$\left[2R\left(p^*_{n^c}\right)R'\left(p^*_{n^c}\right) + ft\varepsilon'\left(p^*_{n^c}\right)\right]\frac{\partial p^*_{n^c}}{\partial t} = f\left[1-\varepsilon\left(p^*_{n^c}\right)\right].$$

Therefore, in the free-entry equilibrium, $\frac{\partial p^*_{n^c}}{\partial t} > 0$, that is, an increase in the transportation cost will increase equilibrium price and hence equilibrium price elasticity of demand $\left(\frac{\partial \varepsilon\left(p^*_{n^c}\right)}{\partial t} > 0\right)$ and industry total revenue $\left(\frac{\partial R\left(p^*_{n^c}\right)}{\partial t} > 0\right)$.

Alternatively, we can use (8) and (11) to substitute out $\left[1-\varepsilon\left(p^*_{n^c}\right)\right]$. The result is $n^c = \frac{R\left(p^*_{n^c}\right)}{f}$. Differentiating the both sides uncovers the following relationship:

$$\frac{\partial n^c}{\partial t} = \frac{R'\left(p^*_{n^c}\right)}{f}\frac{\partial p^*_{n^c}}{\partial t} > 0.$$

Therefore, an increase in the transportation cost will increase the level of entry. As an active firms' revenue is solely determined by f, $\frac{\partial \Pi\left(p^*_{n^c}\right)}{\partial t} = 0$.

4.3 Model outcomes

The above discussion completes our analysis of model outcomes. For a given market, only the fixed cost of entry and the transportation cost are exogenous. Important endogenous variables include, the level of entry n^c, market price $p^*_{n^c}$, price elasticity of demand $\varepsilon\left(p^*_{n^c}\right)$ and industry total revenue $R\left(p^*_{n^c}\right)$. The last three always change in the same direction. We summarize the comparative statics results in the following Proposition.

Proposition 1 *Comparative statics in free entry.*

1. *Entry n^c* decreases, *while price $p^*_{n^c}$, price elasticity of demand $\varepsilon\left(p^*_{n^c}\right)$, and industry total revenue $R\left(p^*_{n^c}\right)$* increase *in the fixed cost f. I.e.,* $\frac{\partial n^c}{\partial f} < 0$, $\frac{\partial p^*_{n^c}}{\partial f} > 0$, $\frac{\partial \varepsilon\left(p^*_{n^c}\right)}{\partial f} > 0$, $\frac{\partial R\left(p^*_{n^c}\right)}{\partial f} > 0$.

2. *Entry n^c, price $p^*_{n^c}$, price elasticity of demand $\varepsilon\left(p^*_{n^c}\right)$, and industry total revenue $R\left(p^*_{n^c}\right)$* increase *in the transportation cost t. I.e.,* $\frac{\partial n^c}{\partial t} > 0$, $\frac{\partial p^*_{n^c}}{\partial t} > 0$, $\frac{\partial \varepsilon\left(p^*_{n^c}\right)}{\partial t} > 0$, $\frac{\partial R\left(p^*_{n^c}\right)}{\partial t} > 0$.

These results are all in line with the standard circular city model except for price elasticity of demand which the standard model does not address. Therefore, on the one hand, our generalization lends support to the use of unit-demand as far as signs of changes are concerned. However, it is worth noting that the current model is much more flexible and better suited to markets in which consumer demand exhibits dependency on price. On the other hand, these results confirm that incorporating general demand functions into the standard models does not result in "discontinuity" in modeling. The current model retains the essence of spatial models and is well suited for analyzing markets with localized competition.

5 Welfare

In addition to understanding market competition, spatial models are also widely used to answer welfare questions. In this section, we compare the number of firms under free entry with two different welfare benchmarks, a first-best benchmark in which the social planner chooses both the level of entry and the prices charged by firms, and a second-best benchmark in which the social planner can only control the level of entry, but not prices. We ask whether there is always excess entry into the market as it is the case in models with completely inelastic demand. If not, how do changes in the exogenous variables affect market efficiency?

In contrast to models with completely inelastic demand, we have to consider prices in our welfare analysis as they have an impact on the quantity purchased and hence on welfare. We define social welfare as the sum of consumer surplus and industry profits:

$$W = V + \int_p^{\hat{p}} q\left(s\right) ds - 2n \int_0^{\frac{1}{2n}} tx \; dx + pq\left(p\right) - nf. \tag{13}$$

5.1 First-best welfare

We start with the first-best benchmark, in which the social planner maximizes total welfare with respect to p and n. From (13), we see that the optimal price is equal to marginal cost, $p = 0$. Inserting this into equation (13) yields

$$W = V + \int_0^{\hat{p}} q(s)\,ds - 2n \int_0^{\frac{1}{2n}} tx\,dx - nf. \tag{14}$$

The problem for the social planner is then identical to the case with completely inelastic demand, hence reduced to a trade-off between transportation costs and fixed costs. The optimal number of entrants is

$$n^f = \sqrt{\frac{t}{4f}}. \tag{15}$$

Proposition 2 *There is excess entry if $\varepsilon(p^*_{n^c}) < \frac{3}{4}$, insufficient entry if $\varepsilon(p^*_{n^c}) > \frac{3}{4}$, and optimal entry if $\varepsilon(p^*_{n^c}) = \frac{3}{4}$.*

Proposition 2 can easily be derived by comparing equations (11) and (15). This proposition provides conditions for the existence of excessive, insufficient, and optimal entry. If the equilibrium demand elasticity is sufficiently low we get excess entry as in the standard model wherein price elasticity is 0. If, on the other hand, equilibrium demand elasticity exceeds $\frac{3}{4}$, there is insufficient entry into the market.

However, the equilibrium demand elasticity is endogenous in this model. Thus, our aim is to state welfare results in terms of exogenous parameters. Note that for a given demand function, $\varepsilon(p)$ is uniquely identified by p. Let p^{**} be defined as the price at which $\varepsilon(p^{**}) = \frac{3}{4}$. By (12),

$$\bar{f}(t) := \frac{1}{t}\frac{R^2(p^{**})}{1-\varepsilon(p^{**})} = \frac{4R^2(p^{**})}{t} \tag{16}$$

is exactly the fixed cost that would result in $p^*_{n^c} = p^{**}$ for a given t. Similarly, $\bar{f}^{-1}(f) := \frac{4R^2(p^{**})}{f}$ is the inverse of (16) which gives the level of transportation cost at which $p^*_{n^c} = p^{**}$ for a given f.

Proposition 3 *Welfare results with respect to the first-best benchmark: For a given demand function $q(p)$, let the constant $\tilde{t}$ be defined as $\tilde{t} := 8R(p^{**})$.*

1. *When $t < \tilde{t}$, there is excess entry for all $f \in (0, F(t)]$;*

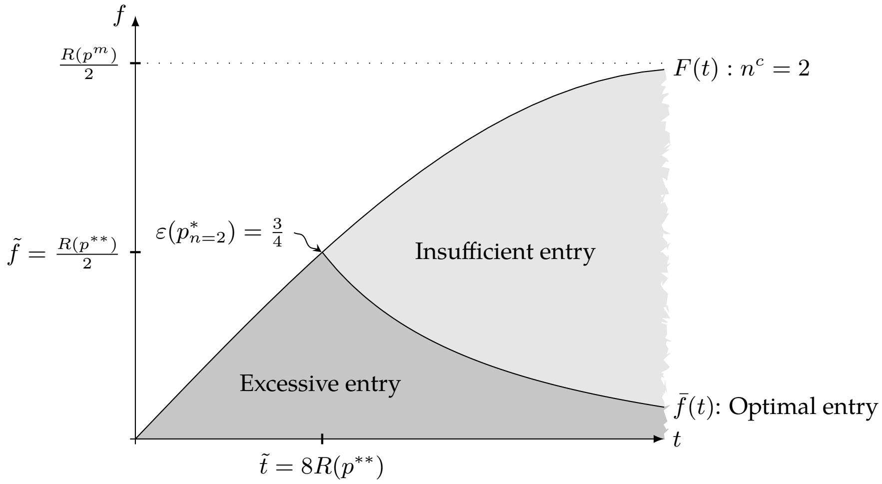

Figure 1: Excessive, optimal, or insufficient entry for relevant parameters

2. *When $t = \tilde{t}$, there is excess entry if $f \in (0, F(t))$ and optimal entry if $f = F(t)$;*

3. *When $t > \tilde{t}$, there is excess entry if $f \in \left(0, \bar{f}(t)\right)$, insufficient entry if $f \in (\bar{f}(t), F(t)]$ and optimal entry if $f = \bar{f}(t)$.*

Proof. *See Appendix A.3.1.* ■

Alternatively, we can differentiate cases by the level of fixed cost. For a given f, the relevant interval of the transportation cost is $t \geq F^{-1}(f)$ where $F^{-1}(f)$ is the inverse of $F(t)$. $F^{-1}(f)$ exists because, for a given demand function, $F(t)$ is strictly increasing in t. The following is a corollary of Proposition 3.

Corollary 1 *Welfare results with respect to the first-best benchmark: For a given demand function $q(p)$, let the constant $\tilde{f}$ be defined as $\tilde{f} := \frac{R(p^{**})}{2}$.*

1. *When $f > \tilde{f}$, there is insufficient entry for all $t \geq F^{-1}(f)$;*

2. *When $f = \tilde{f}$, then there is insufficient entry if $t > F^{-1}(f)$ and optimal entry if $t = F^{-1}(f)$;*

3. *When $f < \tilde{f}$, there is excess entry if $t \in [F^{-1}(f), \bar{f}^{-1}(f))$, insufficient entry if $t > \bar{f}^{-1}(f)$ and optimal entry if $t = \bar{f}^{-1}(f)$.*

Proof. *See Appendix A.3.2.* ■

Proposition 3 and Corollary 1 state the same results from different perspectives. For a given demand function, whether market entry is excessive, optimal or insufficient depends on the transportation cost and the fixed cost. In particular, when the fixed cost of entry is low, there is always excess entry while a high transportation cost provides the possibility of insufficient entry. As the transportation cost increases, $\bar{f}(t)$ decreases and $F(t)$ increases, and hence the interval $(\bar{f}(t), F(t)]$ expands. This means, the higher the transportation cost, the more likely entry is insufficient.

Figure 1 presents the results graphically. The horizontal axis represents the transportation cost and the vertical one the fixed cost of entry. The relevant parameter space is $\{(t, f) | t > 0, 0 < f < F(t)\}$. When $t = \tilde{t}$, $F(t)$ and $\bar{f}(t)$ intersect, and $F(\tilde{t}) = \bar{f}(\tilde{t}) = \tilde{f} = \frac{R(p^{**})}{2}$. In this case, there are exactly two active firms under free entry and at the same time equilibrium price elasticity is $\frac{3}{4}$, and hence entry is socially optimal. If $f = \bar{f}(t)$, then $p^*_{n^c} = p^{**}$ and market entry is optimal. In other cases, whether entry is excessive or insufficient depends on the combination of fixed and transportation costs falls into which region.[21]

5.2 Second-best welfare

In this part, we determine the efficient number of firms under oligopolistic pricing, that is, prices are set according to equation (8). The number of firms (n^s) that maximize welfare is then implicitly given by

$$\frac{n^s t \left[1 - \varepsilon\left(p^*_{n^s}\right)\right] \varepsilon\left(p^*_{n^s}\right) q\left(p^*_{n^s}\right)}{n^s \left[1 - \varepsilon\left(p^*_{n^s}\right)\right] q\left(p^*_{n^s}\right) + t\varepsilon'\left(p^*_{n^s}\right)} + \frac{t}{4} = (n^s)^2 f. \tag{17}$$

A general comparison between free entry and the second-best entry is not attainable.[22] Nevertheless, it is possible to show that entry can also be insufficient compared to the second-best welfare benchmark.

We note that the second-best benchmark level of entry is higher than the first-best level. The intuition is straightforward. If the product price cannot be regulated, compared to the first best level of entry, having more firms in the market results in lower

[21]The shape of $\bar{f}(t)$ can be easily seen form (16) and $F(t)$ is bounded by $\frac{R(p^m)}{2}$. To better present the results, the diagram is not drawn to scale.

[22]In applications, based on the amount of information available, one can either estimate, or make additional assumptions on the demand function in question.

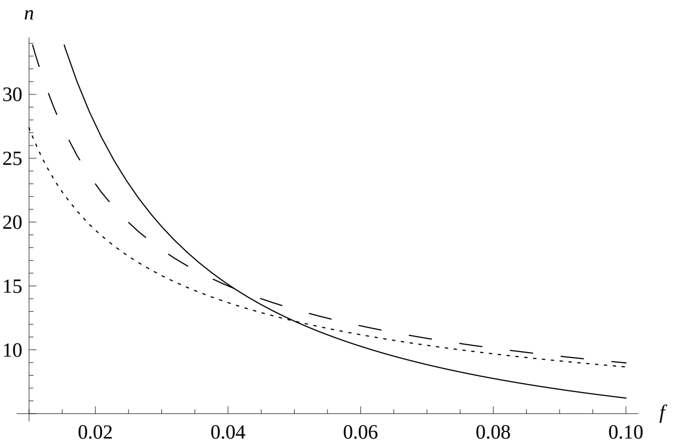

Figure 2: Equilibrium (solid curve) vs. first-best (pointed curve) vs. second-best (dashed curve) entry

equilibrium prices, and hence increases consumption efficiency. This can be seen from equation (17). The last two terms correspond to the first-best condition (15). As the first term is positive it follows that second-best entry is larger than first-best level of entry ($n^s > n^f$). The following proposition then states a sufficient, but not necessary condition, for insufficient entry compared to a second-best benchmark.

Proposition 4 *Entry is insufficient compared to the second-best benchmark if it is insufficient compared to the first-best benchmark.*

The proposition states that whenever entry is insufficient compared to the first-best benchmark, it is also insufficient compared to the second-best benchmark. The opposite, however, is not true. There can be cases where entry is excessive compared to the first-best benchmark but insufficient from a second-best perspective. We illustrate this point by turning to an example with a linear demand function. We use $q(p) = 10(\frac{1}{2} - p)$ and set transportation costs t to 10. We solve numerically for the free-entry equilibrium, the first-best and the second-best level of entry. Figure 2 shows the results of this numerical analysis. While entry is insufficient from a first-best perspective for fixed costs larger than roughly 0.05 it is already insufficient for values larger than 0.04 from a second-best perspective.

In summary, there are more combinations of transportation cost and fixed cost that fall into the category of insufficient entry than those in Figure 1 if the second best benchmark is used.

6 Concluding remarks

In this paper, we incorporated demand functions whose absolute value of price elasticity increases in price into the standard circular city model of product differentiation. We have shown that a unique symmetric price equilibrium exists for any finite number of firms. The proof of this result suggests that our main assumption cannot be significantly relaxed without sacrificing model tractability.[23] This framework can be used to investigate to what extent previous results depend on the assumption of inelastic demand.[24]

The current model is proposed for markets where competition is local and the quantity a consumer demands varies in price. Our analysis shows that after considering price-dependent demand, market entry in spatial models, like in all other models of horizontal product differentiation, can be excessive, insufficient, or optimal depending on model parameters. In this sense, our approach bridges the gap between spatial models and the others in a natural way.[25]

Finally, we think that the current framework and results are not just of theoretic interest. First, our approach allows to derive true policy implications. Second, it may also be useful to empirical researchers who want to investigate whether a specific market contains too much or too few diversity. Our paper offers several hypotheses in this respect. For instance, the scope for too few diversity is the larger the more picky consumers are (that is, the higher are the transportation costs). Furthermore, with sufficient data, one may be able to calibrate a demand function of the form of (4). Since the number of firms is observable, given one of the two cost parameters, our model provides an estimate for the other.

[23] In light of the discussion in Section 3.3, the limit of the main assumption that still ensures the existence of a symmetric price equilibrium might be (-1)-concavity. See footnote 17.

[24] For instance, the results presented in Madden and Pezzino (2011), as well as in Matsumura and Matsushima (2010) and in Suleymanova and Wey (2011).

[25] Calvó-Armengol and Zenou (2002) show that entry need not be excessive when consumers are located far away from firms. However, to maintain this pattern, the distribution of consumers in their model has to be accordingly adjusted each time a new firm enters. In the context of product differentiation, consumers' preferences are not likely to change instantaneously whenever a new variety is offered.

A Appendix

A.1 Proof of Theorem 1

As discussed in Section 3.3, we only have to show that all firms charging the price p^* which is defined by (8) is an equilibrium. Given that, the existence and uniqueness of a symmetric price equilibrium follow directly from Lemma 2. We note that, in this way, we prove existence by construction. We carry out the main task in two steps. Consider a firm, say i, and suppose all other firms are charging p^*. First, we show that when $\bar{x}_i < \frac{1}{n}$ (i.e., no leapfrogging), firm i's profit (6) is strictly quasiconcave in its own price p_i and that $p_i = p^*$ is its best response. Second, we show that $\bar{x}_i \geq \frac{1}{n}$ (i.e., leapfrogging) is *impossible*.

A.1.1 Quasiconcavity

Let $p_o = p^* \in (0, p^m)$. Then, first order derivative (7) becomes

$$\Pi_i'(p_i) = -\frac{q(p_i)}{t}\left[\Delta(p_i) - [1-\varepsilon(p_i)]\int_{p_i}^{p^*} q(s)\,ds\right], \tag{18}$$

where $\Delta(p_i)$ is defined in equation (10). In the proof of Lemma 2 we also established that $\Delta(p_i)$ is strictly increasing in $[0, p^m]$ and obtains a unique root at $p_i = p^*$. This implies $\Delta(p_i) < 0$ in $[0, p^*)$ and $\Delta(p_i) > 0$ in $(p^*, p^m]$. Consider now the part in the square brackets in (18). Since $[1-\varepsilon(p_i)] \geq 0$ for all $p_i \in [0, p^m]$ while $\int_{p_i}^{p^*} q(s)\,ds > 0$ for $p_i \in [0, p^*)$ and $\int_{p_i}^{p^*} q(s)\,ds < 0$ for $p_i \in (p^*, p^m]$, we have

$$\Delta(p_i) - [1-\varepsilon(p_i)]\int_{p_i}^{p^*} q(s)\,ds \begin{cases} < 0 & \text{if } p_i \in [0, p^*) \\ = 0 & \text{if } p_i = p^* \\ > 0 & \text{if } p_i \in (p^*, p^m] \end{cases}.$$

As $-\frac{q(p_i)}{t} < 0$ for $p_i \in [0, p^m]$,

$$\Pi_i'(p_i) \begin{cases} > 0 & \text{if } p_i \in [0, p^*) \\ = 0 & \text{if } p_i = p^* \\ < 0 & \text{if } p_i \in (p^*, p^m] \end{cases}. \tag{19}$$

We note that the profit function (6) is continuous in p_i, in particular, when $p_o = p^* \in (0, p^m)$. Given (19), (6) is strictly quasiconcave on $[0, p^m]$. Note also that any price above p^m is dominated by p^m. Hence, the best response of firm i when all other firms are charging p^* is p^*.[26]

A.1.2 Impossibility of leapfrogging

Here we show that when all other firms are charging p^*, it is impossible for firm i to set a price such that its market share jumps as a result of the demand coming from i's immediate neighbors' "backyard" consumers. First note that for firm i to succeed in this way, p_i has to be low enough to attract consumers with a distance further than $\frac{1}{n}$. Recall that firm i can be located at 0 without loss of generality, and consider a consumer who is located at $\frac{1}{n}$. By (1), for a $p_i \in [0, p^*]$ to leapfrog, the following condition has to hold.

$$\int_{p_i}^{\hat{p}} q(s)ds - \frac{t}{n} \geq \int_{p^*}^{\hat{p}} q(s)ds \Leftrightarrow \int_{p_i}^{p^*} q(s)ds \geq \frac{t}{n}. \tag{20}$$

The objective now is to show there is no such p_i.

To this aim, we construct the following auxiliary demand function. Given that $p^*$is well defined, let $q^* := q(p^*)$, $\varepsilon^* := \varepsilon(p^*)$, and

$$\varphi := q^* \cdot (p^*)^{\varepsilon^*}.$$

Consider the following demand function which has a constant elasticity $\varepsilon^* \in (0, 1)$ (Lemma 2)

$$q^\dagger(p) = \varphi p^{-\varepsilon^*}. \tag{21}$$

Note that (21) also passes through the point (p^*, q^*) as the original demand function $q(p)$ does. Consumer surplus associated with a constant-price-elasticity demand function is easy to evaluate: for[27] $p_i \in [0, p^*]$,

$$\int_{p_i}^{p^*} \varphi s^{-\varepsilon^*} ds = \varphi \frac{(p^*)^{1-\varepsilon^*} - p_i^{1-\varepsilon^*}}{1 - \varepsilon^*}. \tag{22}$$

[26] Whether or not firm i's demand is strictly positive at $p_i = p^m$, that is, whether or not $\bar{x}_i > 0$ is irrelevant. In either case, it pays for firm i to decrease price. The intuition is that p^m is optimal only for a given number of consumers. If reducing price has additional benefits, such as more consumers, there is incentive to charge a lower price.

[27] We can allow for $p_i = 0$ because $\lim_{p_i \to 0^+} \int_{p_i}^{p^*} \varphi s^{-\varepsilon^*} ds$ converges.

Using (8), we have

$$\int_{p_i}^{p^*} \varphi s^{-\varepsilon^*} ds = q^* \left(p^*\right)^{\varepsilon^*} \frac{\left(p^*\right)^{1-\varepsilon^*} - p_i^{1-\varepsilon^*}}{p^* q^* \frac{n}{t}} = \frac{t}{n}\left(1 - \left(\frac{p_i}{p^*}\right)^{1-\varepsilon^*}\right).$$

Because $0 < 1 - \varepsilon^* < 1$ and $0 \leq p_i \leq p^*$,

$$\int_{p_i}^{p^*} \varphi s^{-\varepsilon^*} ds \leq \frac{t}{n}, \text{ for all } p_i \in [0, p^*]. \tag{23}$$

We note that $q^\dagger(p) = \varphi p^{-\varepsilon^*}$ has a constant elasticity ε^* while $q(p)$ obtains elasticity ε^* at the point (p^*, q^*) but strictly lower elasticities when price decreases. This means, for the same percentage decrease in price, with $q(p)$ and $q^\dagger(p)$ starting out at the same point (p^*, q^*), $q(p)$ increases strictly less than $q^\dagger(p)$ does.[28] Therefore, for all $p_i \in [0, p^*]$,

$$q(p) < \varphi p^{-\varepsilon^*} \Rightarrow \int_{p_i}^{p^*} q(s) ds < \int_{p_i}^{p^*} \varphi s^{-\varepsilon^*} ds.$$

By condition (23), relation (24) follows.

$$\int_{p_i}^{p^*} q(s) ds < \frac{t}{n} \text{ for all } p_i \in [0, p^*]. \tag{24}$$

Now we are ready to discuss the (im)possibility of leapfrogging. To leapfrog its neighbors who are charging the symmetric equilibrium price p^*, firm i has to set a price $p_i \in [0, p^*]$ such that condition (20) holds. In contrast, we just established that for all prices in this interval the converse of that condition, i.e., (24), holds. Therefore, leapfrogging is not possible when other firms are charging the candidate equilibrium price.

In summary, we have shown p^* is the unique best response for a firm when all other firms charging p^*. As firm i is chosen without loss of generality, $p_{i=1,2,\ldots,n} = p^*$ is a price equilibrium for any finite ($n \geq 2$) number of firms, and therefore (8) becomes a necessary and sufficient condition for any symmetric price equilibrium. By Lemma 2 such a p^* exists and is unique for any finite ($n \geq 2$) number of firms, and thus the theorem is proved.

[28] Formally, consider the function $\beta(p) = \ln \varphi p^{-\varepsilon^*} - \ln q(p)$ in the interval $(0, p^*]$. Obviously, $\beta(p^*) = 0$. Moreover, $p\beta'(p) = -\varepsilon^* + \varepsilon(p) < 0$, for $p < p^*$. This implies $\ln \varphi p^{-\varepsilon^*} > \ln q(p)$ and hence $\varphi p^{-\varepsilon^*} > q(p)$. As $q(p)$ is bounded, this relation holds when $p = 0$ (if we allow for extended real numbers).

A.2 Proof of Lemma 3

1. Keep t constant and perform differentiation to both sides of (8) with respect to n,

$$\begin{aligned} R'(p^*)\frac{\partial p^*}{\partial n} &= -\frac{t[1-\varepsilon(p^*)]}{n^2} - \frac{t}{n}\varepsilon'(p^*)\frac{\partial p^*}{\partial n} \\ \Rightarrow \frac{\partial p^*}{\partial n} &= -\frac{t[1-\varepsilon(p^*)]}{n^2 q(p^*)[1-\varepsilon(p^*)] + nt\varepsilon'(p^*)}. \end{aligned}$$

Since $[1-\varepsilon(p^*)] > 0$ by Lemma 2 and $\varepsilon'(p^*) > 0$ by Assumption 1, $\frac{\partial p^*}{\partial n} < 0$. Moreover,

$$\begin{aligned} \frac{\partial \varepsilon(p^*)}{\partial n} &= \varepsilon'(p^*)\frac{\partial p^*}{\partial n} < 0, \quad \frac{\partial R(p^*)}{\partial n} = R'(p^*)\frac{\partial p^*}{\partial n} < 0, \text{ and} \\ \frac{\partial \Pi(p^*)}{\partial n} &= \frac{\partial}{\partial n}\left(\frac{R(p^*)}{n}\right) = \frac{n\frac{\partial R(p^*)}{\partial n} - R(p^*)}{n^2} < 0. \end{aligned}$$

2. This part follows from the first part of Lemma 3 and the fact that t and n appear as a quotient in (8).

A.3 Proof of Proposition 3 and Corollary 1

A.3.1 Proof of Proposition 3

Let t be fixed and start with $f = F(t)$. By Proposition 1, when f decreases, $\varepsilon(p^*_{n^c})$ decreases from $\varepsilon(p^*_{n=2})$. If $\varepsilon(p^*_{n=2}) < \frac{3}{4}$ then it is clear that all equilibrium elasticities will be less than $\frac{3}{4}$, and hence entry will always be excessive (Proposition 2).

If, on the other hand, $\varepsilon(p^*_{n=2}) > \frac{3}{4}$, then entry is insufficient at $f = F(t)$. As f decreases, so does $\varepsilon(p^*_{n^c})$ and eventually $\varepsilon(p^*_{n^c})$ will reach $\frac{3}{4}$ because of continuity. When $\varepsilon(p^*_{n^c}) = \frac{3}{4}$, entry is optimal, and $f = \bar{f}(t)$ by the definition of $\bar{f}(t)$. Therefore, for $f \in (\bar{f}(t), F(t)]$, entry is insufficient and for $f < \bar{f}(t)$, excessive entry results.

If $\varepsilon(p^*_{n=2}) = \frac{3}{4}$, entry is optimal at $f = F(t)$. A decrease in f brings down the equilibrium elasticity and hence entry is excessive for all $f < F(t)$. We now only have to show the relation between $\varepsilon(p^*_{n=2})$ and $\tilde{t}$.

As $\varepsilon(p^*_{n=2})$ is the equilibrium price elasticity for a given number of firms ($n = 2$), by (8) we have $t = \frac{2R(p^*_{n=2})}{1-\varepsilon(p^*_{n=2})}$. Using Lemma 3 and the observation that $R(p^*_{n=2})$ is bounded by $R(p^m)$, it is easily checked that as t increases from 0, $\varepsilon(p^*_{n=2})$ increases from 0 and eventually to 1. For a given demand function, $\varepsilon\left(p^*_{n=2}\right) = \frac{3}{4}$ if and only if the transportation cost is $\tilde{t} = 8R\left(p^{**}\right)$. Therefore, when $t > \{=, <$, resp.$\}$ $\tilde{t}$, $\varepsilon\left(p^*_{n=2}\right) > \{=, <$, resp.$\}$ $\frac{3}{4}$. Proposition 3 then follows.

A.3.2 Proof of Corollary 1

First, we note that $F\left(\tilde{t}\right) = \frac{8R(p^{**})}{4}\left[1 - \frac{3}{4}\right] = \frac{R(p^{**})}{2} = \tilde{f}$. If $f > F\left(\tilde{t}\right)$, to ensure there are at least two firms enter, $t \geq F^{-1}\left(f\right) > \tilde{t}$. From the Proof of Proposition 3, we know in this case $\varepsilon\left(p^*_{n=2}\right) > \frac{3}{4}$. By Proposition 1, equilibrium elasticity increases from $\varepsilon\left(p^*_{n=2}\right)$ as t increases form $F^{-1}\left(f\right)$. Therefore, according to Proposition 2, there is always insufficient entry.

If $f < F\left(\tilde{t}\right)$, then $F^{-1}\left(f\right) < \tilde{t}$ and hence $\varepsilon\left(p^*_{n=2}\right) < \frac{3}{4}$. When t increases from $F^{-1}\left(f\right)$, $\varepsilon\left(p^*_{n^c}\right)$ will increase from $\varepsilon\left(p^*_{n=2}\right)$ and reach $\frac{3}{4}$ at $t = \bar{f}^{-1}\left(f\right) = \frac{4R^2(p^{**})}{f}$. For $t > \bar{f}^{-1}\left(f\right)$, $\varepsilon\left(p^*_{n^c}\right) > \frac{3}{4}$. Therefore, there is excess entry if $t \in \left[F^{-1}\left(f\right), \bar{f}^{-1}\left(f\right)\right)$, insufficient entry if $t > \bar{f}^{-1}\left(f\right)$ and optimal entry if $t = \bar{f}^{-1}\left(f\right)$. This also proves the $f = F\left(\tilde{t}\right)$ case.

References

Anderson, S. P. and de Palma, A. (2000). From local to global competition. *European Economic Review*, 44(3):423–448.

Anderson, S. P., de Palma, A., and Thisse, J.-F. (1992). *Discrete Choice Theory of Product Differentiation*. MIT Press.

Boeckem, S. (1994). A generalized model of horizontal product differentiation. *Journal of Industrial Economics*, 42(3):287–298.

Calvó-Armengol, A. and Zenou, Y. (2002). The importance of the distribution of consumers in horizontal product differentiation. *Journal of Regional Science*, 42(4):793–803.

Caplin, A. and Nalebuff, B. (1991). Aggregation and imperfect competition: on the existence of equilibrium. *Econometrica*, 59(1):25–59.

d'Aspremont, C., Gabszewicz, J. J., and Thisse, J.-F. (1979). On Hotelling's "stability in competition". *Econometrica*, 47(5):1145–1150.

Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. *American Economic Review*, 67(3):297–308.

Gu, Y. and Wenzel, T. (2009). A note on the excess entry theorem in spatial models with elastic demand. *International Journal of Industrial Organization*, 27(5):567–571.

Hotelling, H. (1929). Stability in competition. *Economic Journal*, 39:41–57.

Janssen, M. C. W., Karamychev, V. A., and van Reeven, P. (2005). Multi-store competition: Market segmentation or interlacing? *Regional Science and Urban Economics*, 35(6):700–714.

Madden, P. and Pezzino, M. (2011). Oligopoly on a salop circle with centre. *The B. E. Journal of Economic Analysis & Policy*, 11(1):Article 2.

Matsumura, T. (2000). Entry regulation and social welfare with an integer problem. *Journal of Economics*, 71(1):47–58.

Matsumura, T. and Matsushima, N. (2010). When small firms fight back against large firms in R&D activities. *The B. E. Journal of Economic Analysis & Policy*, 10(1):Article 81.

Matsumura, T. and Okamura, M. (2006a). Equilibrium number of firms and economic welfare in a spatial price discrimination model. *Economics Letters*, 90(1):396–401.

Matsumura, T. and Okamura, M. (2006b). A note on the excess entry theorem in spatial markets. *International Journal of Industrial Organization*, 24(5):1071–1076.

Neven, D. J. (1986). On Hotelling's competition with non-uniform customer distributions. *Economics Letters*, 21:121–126.

Norman, G. and Thisse, J.-F. (1996). Product variety and welfare under tough and soft pricing regimes. *Economic Journal*, 106(434):76–91.

Rath, K. and Zhao, G. (2001). Two stage equilibrium and product choice with elastic demand. *International Journal of Industrial Organization*, 19(9):1441–1455.

Salop, S. (1979). Monopolistic competition with outside goods. *Bell Journal of Economics*, 10:141–156.

Shilony, Y. (1981). Hotelling's competition with general customer distributions. *Economics Letters*, 8:39–45.

Spence, M. (1976). Product selection, fixed costs, and monopolistic competition. *Review of Economic Studies*, 43(2):217–35.

Suleymanova, I. and Wey, C. (2011). Bertrand competition in markets with network effects and switching costs. *The B. E. Journal of Economic Analysis & Policy*, 11(1):Article 56.

Tirole, J. (1988). *The Theory of Industrial Organization*. MIT Press.

Vickrey, W. S. (1964). *Microstatics*. Harcourt, Brace and World, New York.

Ziya, S., Ayhan, H., and Foley, R. (2004). Relationships Among Three Assumptions in Revenue Management. *Operations Research*, 52(5):804–809.

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