Injunctions Against
False Advertising

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Abstract

Rules of consumer protection or fair competition can be publicly or privately enforced. We consider the possibility of false advertising by a firm in duopolistic competition where consumers can be distinguished according to whether or not they form rational beliefs about the trustworthiness of advertising claims. We compare private and public law enforcement in the form of the demand for injunctions against false advertising. From a welfare perspective, we show that it can be optimal either to have the private entity (the competitor/a consumer protection agency) or the government agency as plaintiff, where the optimal regime depends on the share of naïve consumers and the level of trial costs in a non-trivial way.

Keywords: injunction suits, false advertising, law enforcement, naïve consumers, product differentiation.

JEL classification: K41, K42, L13, L15.

1   Introduction

We analyze injunction suits in the context of false advertising, a prominent field in consumer protection and unfair competition law. Advertising is a fundamental strategic variable for firms, which they use in an attempt to gain a competitive edge over their competitors. To this end, firms advertise product attributes such as durability, effectiveness, (environmental) safety, or origin to convince consumers of the high quality of their products. For consumers, the value of the information conveyed by advertisements depends on its trustworthiness, because firms may actively try to misled consumers thereby harming them as well as competitors. Consequently, in most countries, firms are restricted in that they are not allowed to make incorrect claims or use misleading advertising. In particular, courts can order injunctions requiring the firm to stop its misleading advertising campaign. However, the rules specifying which parties are allowed to file injunction suits or take other actions against false advertising differ across countries.

The field of unfair competition law, including rules on misleading advertising, indeed is an illustrious example for either public or private law enforcement or combinations of the two forms. For example, in the European Union, Directive 2005/29/EC sets the rules concerning misleading advertising in the business to consumer relation. Article 11 requires the member states to "ensure adequate and effective means" to enforce firms' compliance in the interest of consumers and explicitly stresses the role of persons and institutions with legitimate interest in bringing forth claims. However, the mode of implementation is left to national governments and in particular allows for either the possibility of private parties to directly initiate court proceedings or the involvement of a specialized administrative authority that may start legal proceedings (or combinations thereof). This means that, in principle, government agencies as well as competitors or consumer protection agencies may take legal action against misleading advertising in Europe where the European Commission points out that the implementation of the directive differs across the member states (see, European Commission, 2012). In some countries, it is mainly public authorities that can take action against rogue traders (e.g., in France, Italy, and the United Kingdom), whereas Austria and Germany are prime examples of countries relying on private law enforcement by competitors, consumer protection agencies, or industry organizations. At the same time, an intense discussion is going on in Germany whether to

complement consumer protection by a government agency.[1] In comparison, in the United States, the Federal Trade Commission (FTC) and its Division of Advertising Practices enforce the truth-in-advertising laws across different media outlets.[2] However, private parties can also bring forth claims against false advertising on their own (for example based on Section 43(a) of the Lanham Act).[3]

We are interested in the comparison of the incentives for private parties and government agencies to stop false advertising campaigns by filing an injunction suit.[4] With regard to private parties, we focus on competitors and consumer protection agencies as potential plaintiffs.[5] We do this under the assumption that some consumers take all uncontested advertising messages for face value, whereas rational consumers understand the probability that an uncontested advertisement may contain exaggerated claims about a product's quality. Besides a positive description of the incentives to file suit, we also analyze the equilibrium outcomes from a welfare perspective.

Despite the prevalent use of false advertising, firms' possibility to bring cases against their competitors in court, and the attempts by government bodies and consumer protection agencies to protect consumers from fraud and deception in the marketplace,[6] there is

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[1] The topic has for example been discussed at a recent conference hosted by the Federal Ministry of Justice and Consumer Protection, see Schulte-Nölke (2017).

[2] The FTC "authorizes the filing of a complaint when it has 'reason to believe' that the law has been or is being violated and it appears to the Commission that a proceeding is in the public interest" (`ftc.gov`). In addition, alongside its authority to investigate law violations by individuals and businesses, the FTC also has federal rule-making authority to issue industry-wide regulations (e.g., labeling requirements).

[3] For example, in the 2014 case *POM Wonderful LLC v. Coca-Cola Co*, the Supreme Court—even though food and beverage labels are subject to Food and Drug Administration (FDA) regulation—permitted producer POM to proceed with its false advertising claim that "one of Coca-Cola's juice blends mislead consumers into believing the product consists predominantly of pomegranate and blueberry juice when it in fact consists predominantly of less expensive apple and grape juices, and that the ensuing confusion causes POM to lose sales" (see `https://www.supremecourt.gov/opinions/13pdf/12-761_6k47.pdf`, p. 1). This case also highlights another important aspect: the potential difference between a regulator's and a competitor's incentive to file suit. As a matter of fact, Coca-Cola's labeling complied with FDA requirements.

[4] Depending on the country considered, a government agency may not act as a plaintiff but may itself be in a position to order firms to stop misleading advertising campaigns after an investigation. In this case, our results are to be understood as a comparison between a private party's incentives to file an injunction suit and the incentives for the government agency to start an investigation.

[5] For instance, in Germany, the *Act Against Unfair Competition (UWG)* explicitly mentions entities acting to protect consumer interest and competitors as potential plaintiffs. Indeed, these groups are responsible for a large share of injunction suits in Germany.

[6] In the US, the range of products for which firms have invented deceptive advertising campaigns, which were later challenged by the FTC, includes such diverse products as dog food (see `https://www.ftc.gov/news-events/press-releases/2016/08/mars-petcare-settles-false-advertising-charges-related-its`), allegedly mosquito-repellent wristbands (see `https://www.ftc.gov/news-events/press-releases/2015/02/ftc-charges-company-`

only limited theoretical research on the economic effects and implications of false advertising that explicitly takes into account law enforcement. In this paper, we contribute to the literature in two important ways. First, we take the fact of differences in national legislation as a motivation to analyze the incentives of different parties (either a competitor interested in profit maximization, a consumer protection agency interested in consumer welfare, or a government agency interested in maximizing ex-post social welfare) to go to court to effect an injunction stopping a deceptive advertising campaign. From a social welfare and consumer point of view, these incentives have important implications with regard to who should be given the right to go to court in the first place (to maximize the ex-ante expected surplus). Second, we identify consumer rationality and the level of trial costs as two important factors when it comes to designing an adequate legal framework. Both elements affect behavior of private and public plaintiffs in distinct ways with direct repercussions for social welfare regarding the allocation of the right of action.

We analyze a duopoly market with horizontal product differentiation à la Hotelling (1929). In this market, one firm offers a product of known *standard* quality, whereas the other firm has private information with regard to whether its product quality is *high* or *standard* (which introduces possible vertical product differentiation). The latter firm decides on whether to (possibly falsely) advertise a *high* product quality, and one of the private plaintiffs or a government agency may challenge this claim by filing an injunction suit. Injunction suits result in trial costs which are—in the main part of the paper—

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owner-deceptively-marketing-mosquito), and sports shoes (see `http://www.ftc.gov/news-events/press-releases/2011/09/reebok-pay-25-million- customer-refunds-settle-ftc-charges` and `http://www.ftc.gov/news-events/press-releases/ 2012/05/skechers-will-pay-40-million -settle-ftc-charges-it-deceived`). Moreover, there are a number of pending cases. The most prominent recent case is Volkswagen's deceptive "Clean Diesel" ad campaign, presumably resulting in billions of US\$ in fines and compensations (see, e.g., `https://www.ftc.gov/news-events/press-releases/2016/10/federal-judge-approves-ftc-order-owners-certain-volkswagen-audi`). In Germany, consumer protection agencies took legal action against producers of jam (*Darbo* case) and tea among others for what they considered misleading labeling (see `http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A61998CJ0465` and `http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A62014CJ0195`). In the UK, the Advertising Standards Authority (ASA) stepped in to ban an advert by the supermarket chain Tesco in response to the horsemeat scandal; the advert suggested the problem affected "the whole food industry" (see `http://www.telegraph.co.uk/finance/newsbysector/retailandconsumer/10286180/Tesco-rapped-over-misleading-horsemeat-ads.html`.) In *I ZR 16/14* the German Federal Court (BGH) confirmed the injunction initiated by a competitor against its rival advertising condoms as "made in Germany", because production occurred abroad, and only final control and packaging was done in Germany.

allocated according to the *English* cost allocation rule.[7] In court, the product's true quality is established. With regard to consumers, it is assumed that there is a group of rational consumers, who take into account that advertising may convey wrong information, and a group of naïve consumers, who fully trust commercial ads not challenged in court.

Our results for the comparison of private and public plaintiffs are driven by two main mechanisms. First, there is a fundamental distortion induced by the differences in (expected) product qualities and prices: From a social welfare point of view, consumer choices should be made based on a comparison of (additional) transport costs and (higher) expected product quality. However, the actual decisions in the market equilibrium are distorted for two reasons: First, firms charge different prices, which impact all consumers' purchase decisions. As the firm with the better (expected) quality charges a higher price, this product tends to be underconsumed. Second, there is an effect which works against the price-induced tendency towards underconsumption of the product with the allegedly higher quality: For given prices, naïve consumers consume more of it than rational consumers, because they do not question the advertising message, i.e., they have distorted beliefs. The latter distortion would be eliminated by an injunction suit revealing the true quality of products, whereas the first distortion due to the price difference persists. In consequence, the existence of naïve consumers can reduce incentives for an injunction suit for the government agency, whereas it increases incentives for private parties that are especially hurt by misled naïve consumers (either directly, as consumer rent decreases, or indirectly, as profits decline).

The second important mechanism is the existence of a possible commitment problem of the law enforcer. An injunction can only be filed after an advertisement has been released, whereas the decision on filing a (probably false) advertisement is based on the likelihood of being taken to court. That is, a more aggressive strategy by the plaintiff (a higher probability of filing a lawsuit) is likely to deter false advertisement in the first place. The decision on filing is taken at a point in time when it only affects welfare and profits ex post, and therefore only these repercussions may guide the filing decision. In such a setting, it may be welfare-enhancing to appoint a private plaintiff that is more aggressive than the government agency even if from an ex-post perspective, the private plaintiff will

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[7]The *English* cost allocation rules implies that the party losing in court bears the full trial costs, i.e., it has to reimburse the prevailing party's costs.

file suit, although the costs do not justify the ex-post expected gain in welfare.

We refer to these two main effects that explain our results at different points in the paper.

We find that when trial costs are not too high, there is an equilibrium in mixed strategies in which the firm with the unobservable quality sometimes falsely advertises a high quality, and the plaintiff sometimes files a suit. When trial costs are high, the plaintiff refrains from filing a suit and the firm with the unobservable quality always advertises a high quality (independent of actual quality). With regard to the impact of the share of naïve consumers, it turns out that for given trial costs, a private plaintiff is more likely to file a suit when the share of naïve consumers increases. When the government agency must authorize the filing of a suit, for some intermediate trial costs, a higher share of naïve consumers first increases the likelihood of a trial before it decreases again. A comparison of the incentives to go to court yields that the government agency is more likely to initiate legal action for low shares of naïve consumers and low trial costs; otherwise, the private plaintiff is (weakly) more likely to take legal action. These observations can be related to the first mechanism described above. Whereas a higher share of naïve consumers always yields a larger loss due to uncorrected false advertising for private plaintiffs, the existence of naïve and uninformed consumers may alleviate distortions in the market equilibrium that result from firms' price setting. Accordingly, with social welfare in mind, consumer naïveté may reduce incentives for the government agency to demand an injunction. Comparing the two private plaintiffs, in our duopoly setup it is always the competitor that has more pronounced incentives to file an injunction suit compared to the consumer protection agency.

These positive findings have important normative implications for consumer surplus and ex-ante expected social welfare. In equilibrium, consumers always prefer the plaintiff with the greater incentives to file an injunction suit (the more aggressive plaintiff), that is, the government agency for low shares of naïve consumers and low trial costs and the private plaintiff otherwise. In contrast, social welfare—where in addition to consumer surplus, trial costs and firms' profits are also relevant—can be higher or lower for the more aggressive private plaintiff. We find that expected social welfare is higher for the government agency as plaintiff when the share of naïve consumers and trial costs are low, and when trial costs are (very) high. In the first case, the government agency is

the more aggressive plaintiff yielding a higher level of deterrence of false advertisement and at the same time the private plaintiff has too little incentive to go to court even if only the ex-post gains in expected welfare are taken into account. In the second case of high-trial costs and an intermediate to high share of naïve consumers, a government body would abstain from filing suit and save on the trial costs, whereas the private plaintiff may file an injunction suit. In this case, however, although higher filing rates lead to fewer false advertisement in equilibrium, this advantage of private enforcement is more than offset by the high level of trial costs. In contrast, for low to intermediate trial costs and a sufficiently high share of naïve consumers, expected social welfare is higher under a private plaintiff. In this case, again the more aggressive behavior by the private plaintiff reduces the rate of false advertising, but now the accompanying gain in welfare more than offsets the additional trial costs in comparison to the government agency acting as potential plaintiff. The possibility of this latter result is due to the commitment problem described above as the second main mechanism of our model.

The paper proceeds as follows. In Section 2, we review the related literature. Section 3 presents the model. We derive the equilibrium in Section 4 and compare the incentives to file an injunction suit for the government agency and the two potential private plaintiffs. In Section 5, we describe the implications for social welfare and consumer surplus. Section 6 discusses three extensions. Section 7 concludes.

2   Literature review

Our paper focuses on false advertising regarding a product's quality[8] and law enforcement when advertising is addressed to both rational and naïve consumers.[9]

In an early contribution, Nelson (1974) distinguishes between search and experience goods emphasizing that with search goods, advertising must contain hard facts, whereas

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[8]Some papers on false advertising consider specific practices which are not related to claims about quality. For example, Lazear (1995) considers the strategy of "bait and switch" (the seller falsely claims to provide a good to lure consumers to its store), whereas Armstrong & Chen (2013) investigate the false depiction of offered prices as a discount. Similarly, Deng et al. (2018) investigate firms' incentives to offer false promotions and the impact of consumer sophistication and word-of-mouth.

[9]Advertising regarding product quality may also be considered as a specific form of information transmission, which is part of a more general strand of the literature. Models in this vein consider the possibility of credible information transmission of self-interested parties and sometimes invoke the notion of lying costs. See, for example, Milgrom & Roberts (1986), Kartik et al. (2007), Kartik (2009), and Kamenica & Gentzkow (2011).

for experience goods, the fact of advertisement allows for some inference of true quality. In this vein, he already points to the possibility of deceptive advertising and how its occurrence should depend on the level of law enforcement. Schmalensee (1978) starts from Nelson (1974) and considers advertising expenditures as contributions to a contest in a setting with partly naïve consumers. He shows that—in contrast to the finding in Nelson (1974)—it may be the low-quality firm which advertises more, thereby somehow deceiving consumers. Related to Schmalensee (1978), Mizuno & Hiroyuki (1990) run a simulation model in which consumers are affected by advertising but may or may not be able to learn about true product quality. With consumers being able to learn, misleading advertisement may no longer pose a threat in a long-run equilibrium. Dixit & Norman (1978) assume that advertising leads to an outward shift in demand for a firm and discuss whether the original or the shifted demand curve should be used for evaluating consumer welfare, a discussion which may be related to the discussion of advertisement of facts or misleading advertisement.

More recently, citing Dixit & Norman (1978) as their benchmark model, Glaeser & Ujhelyi (2010) analyze various responses to regulating misinformation (counter-advertising, taxation, and product regulation). The authors assume that consumers naïvely believe all advertising claims (see also Hattori & Higashida, 2012, who consider a duopoly model with horizontally differentiated products; Hattori & Higashida (2014) investigate a duopoly with vertically differentiated products and naïve consumers; Hattori & Higashida (2014) investigate a duopoly with vertically differentiated products and naïve consumers and find that false advertising can increase the degree of price competition). Their results suggest that a certain amount of misinformation about product quality may improve social welfare under imperfect competition. Despite the fact that misinformation tends to reduce consumer surplus (as consumers buy products they would not have purchased in the first place), misinformation mitigates the problem of underconsumption of products resulting from imperfect competition. As a consequence, government regulations that benefit consumers can harm firms and reduce overall social welfare.[10] Different from their study, we allow for different shares of naïve consumers, consider a covered market (i.e., constant demand), and, most importantly, focus on law enforcement against misleading claims.

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[10]Matsumura & Sunada (2013) extend the model of Glaeser & Ujhelyi (2010) by a public firm interested in social welfare and competing with private profit-maximizing firms in the market. They find that the public firm will also make use of persuasive advertising.

As in our model, Nagler (1993) and Hattori & Higashida (2015) relax the assumption of only naïve consumers. Nagler (1993) allows consumers to learn about the trustworthiness of advertising at some costs and establishes that firms may still use misleading advertising. Hattori & Higashida (2015) set up a model of price competition between two firms producing horizontally and vertically differentiated products allowing for misleading advertising. They focus on the allocative implications of misinformation with regulatory policies like advertising taxes, or unit and ad valorem taxes on production being in place. Similar to Glaeser & Ujhelyi (2010), misinformation has two effects which go in opposite directions from a social welfare point of view: Misinformation distorts consumers' decision-making but can correct inefficiencies resulting from the misallocation of goods. It is shown that advertising can create a prisoner's dilemma for firms and reduce overall welfare while benefiting certain consumer groups: smart consumers who are not affected by misinformation and those with a preference for low-quality brands. Different from their study, advertising is not always misleading in our case but can be informative. Furthermore, fully rational consumers in our setup may not know the product quality with certainty but also learn from advertisements, and again we focus on different parties' incentives to take legal action.

The papers discussed so far neglect law enforcement. A different strand of the literature introduces fines for false statements and considers rational consumers and costless advertising. As pointed out in Corts (2013) in a monopoly setting, the existence of fines allows for advertising to become a credible signal of high quality. Depending on the fine, pooling or separating equilibria for advertising will occur.[11] Corts (2014a) introduces the idea that very high fines may be detrimental when firms face uncertainty about product quality and may refrain from advertising altogether. Piccolo et al. (2015) use a duopoly model in which the consumer buys one unit from either of two firms. As in Corts (2013) the type of equilibrium depends on the fine for false statements. The authors emphasize that high fines can hurt the consumer, as the degree of price competition is weakened, and this can more than offset the advantage from buying the higher quality product with a higher probability. In contrast, in the monopoly case, the fine should always guarantee a separating equilibrium from the consumers' point of view.

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[11]Using the same basic setting as in Corts (2013), Corts (2014b) more generally looks at the welfare effects of information about quality.

Closely related to this literature and also to our paper is Rhodes & Wilson (2018). The authors analyze a model in which a regulator is able to punish false claims in a situation with rational consumers. In their setup, false advertising can have a beneficial effect ('price' effect) by counteracting market power, as the high-quality firm will choose a lower price if it cannot signal its type. In consequence, firms favor stricter regulations (higher fines) than consumers do. At the same time, the negative consequences of false advertising are due to the 'persuasion' effect, making consumers misperceive the true quality. The authors provide conditions under which weak, rather than strong, regulation can be optimal for consumers and society due to the positive effects of false advertising in counteracting firms' market power. Different from these studies which mainly focus on a monopoly setup (see Piccolo et al., 2015 for an exception) and assume rational consumers throughout the analysis, we focus on competition and allow for naïve consumers.[12] More importantly however, instead of introducing an expected fine, we assume a different kind of law enforcement by explicitly modeling the decision whether or not to file an injunction suit for different types of plaintiffs.

The topic of different plaintiffs has received only limited attention in the literature on advertising. With regard to competitors as plaintiffs in false advertising claims, Barigozzi et al. (2009) consider comparative advertising. In their model, false comparative advertising allows the competitor to file suit, which may not be possible in the event of false non-comparative statements about product quality. As in Corts (2013), the threat of lawsuits and punishment enables the firm to make credible assertions about its product quality and can therefore facilitate market entry of high-quality firms.[13] The latter finding contrasts with the earlier considerations of Jordan & Rubin (1979) who deliberate on the incentives of different parties to file suit with regard to false advertising. Whereas consumers might not file suit due to a rather low value of the individual claim, the authors indicate that allowing for injunction suits by competitors may be misused as a device to deter market entry especially when claims about quality are difficult to verify. In our setup, we abstract from the possibility of errors in court but also indicate that advertising

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[12] Allowing for naïve and rational consumers has increasingly gained attention in the industrial organization literature. Different from our advertising setup, these models often look at strategies like hidden prices which firms may use to exploit consumer naïveté. See, for example, the early survey by Ellison (2006) or contributions like Gabaix & Laibson (2006) and Heidhues et al. (2017).

[13] Emons & Fluet (2012) also compare non-comparative and comparative advertising and allow for overstatement of product quality in a signaling game.

becomes more credible the more aggressive the potential plaintiff is.

3   The model

We consider a model of horizontal product differentiation à la Hotelling (1929) with two firms, 1 and 2. The firms are located at the extremes of a linear city of unit length, with firm 1 being located at $L_1 = 0$ and firm 2 at $L_2 = 1$. Firm 2 offers a product of normal quality which consumers value at v, $v > 0$. Firm 1 is one of two types. With probability $1/2$ it is of the high type and produces a good of high quality which is valued by consumers at $v + 1$. With probability $1/2$ firm 1 is of normal type and offers a product with normal quality as its competitor. The type of firm 1 is its private information which is not observed by neither consumers prior to purchase nor by firm 2. For both firms and both quality levels, marginal costs of production are normalized to zero.

Consumers of mass one are uniformly distributed along the line. Each consumer buys one unit either from firm 1 or from firm 2, i.e., the market is covered.[14] Consumers know that firm 2 offers a product of normal quality, whereas firm 1 may offer a high-quality product. A consumer located at x who buys from firm i (with $i \in \{1, 2\}$) pays price p_i and bears linear transport costs $|L_i - x|$. We consider two types of consumers. At each point along the line, a share q of consumers is naïve in a sense made precise below (subscript n). The remaining share $1 - q$ of consumers is rational (subscript r). Consumers' belief with regard to the probability that firm 1 offers a product of high quality is denoted by ψ_j, with $0 \leq \psi_j \leq 1$, $j = n, r$.

Knowing its own type, firm 1 has the choice whether or not to (possibly falsely) advertise a high product quality to consumers. Advertising is assumed costless per se.[15] Note that attention will be restricted to equilibria in which a firm serving high quality always advertises and if consumers do not observe advertising activity by firm 1, they expect the firm to offer a product of normal quality (superscript N), i.e., $\psi_n = \psi_r = \psi^N = 0$.[16]

We compare two different scenarios regarding law enforcement: private and public law

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[14]To this end, we make the standard assumption that the valuation v is sufficiently large.

[15]We mainly require that advertisement costs do not depend on the product's true quality. By this assumption, signaling via advertising expenditures is excluded.

[16]This assumption has been employed in other recent contributions on misleading advertising as well (e.g., Corts 2013). By doing so, we exclude the existence of some less plausible but possible equilibria.

enforcement. In both scenarios, in the event that firm 1 advertises a high quality, it may face an injunction suit. In the first scenario, a private entity, in our case the competitor or a consumer protection agency, can initiate the lawsuit. Whereas firm 2 maximizes expected profits, the consumer protection agency maximizes expected consumer surplus taking into account own trial costs. In the second scenario, the decision to file an injunction suit is taken by a government agency which aims at maximizing expected social welfare given the market equilibrium.[17] The overall costs for both parties, plaintiff and defendant, in a lawsuit are denoted by k, $k > 0$, which includes the costs of verifying quality.[18] The cost-sharing rule applied corresponds to the English rule, which means that full trial costs are borne by the losing party.[19] In our setup, both private and public plaintiffs have the same knowledge about the market characteristics, excluding possible information advantages of private parties as discussed in McAfee et al. (2008). This allows us to focus on differences in incentives for filing a lawsuit.

If the claim of high quality is falsified in court advertisements have to be discontinued, and firm 1 bears the full trial costs; in this event, all consumers share the same belief about the probability that firm 1 serves high quality such that $\psi_n = \psi_r = \psi^N = 0$. If the sued firm indeed offers high quality, an injunction will not be granted and the costs k have to be borne by the plaintiff. Because quality has been verified in court, all consumers again share the same belief about the probability that firm 1 serves high quality such that $\psi_n = \psi_r = \psi^H = 1$.[20, 21] When firm 1 advertises a high quality and is not challenged by an injunction suit (superscript A), consumers do not receive any further information

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[17]For the sake of comparability, we assume that the government agency files a lawsuit as do private parties. Alternatively, the government agency may be equipped with the rights to issue an injunction on its own after a corresponding investigation.

[18]Hence, we neglect the possibility of different costs for public and private enforcement. This is to be understood as a simplification of the analysis that allows us to focus on the difference in incentives of potential plaintiffs. Certainly costs could differ leading to corresponding consequences for welfare. For an early discussion of this issue, see Polinsky (1980).

[19]Qualitatively, our results are not affected if we instead assume the American rule for cost sharing, which implies that each party bears its own litigation costs (see Section 6.1).

[20]Given the above description, the scenario with the government agency as the plaintiff is equivalent to the following situation: Instead of filing a case, the government agency itself may act as a ruling body. In this case, the filing decision is replaced by the decision to take up investigations associated with costs k. The results of the investigation are made public and costs k are burdened on firm 1 in the event of a detected misleading advertising.

[21]Note that due to the assumption that the injunction suit precedes consumers' buying decision, no direct harm is incurred by consumers or the competitor in the event of a successful injunction suit. Correspondingly, we abstain from damages payments in this case although the model could be extended to encompass this additional instrument.

in addition to observing an advertisement. In this case, naïve consumers are assumed to fully believe in firm 1's advertising message and hence $\psi_n = \psi_n^A = 1$. In contrast, rational consumers update their beliefs taking into account that the firm may have falsely advertised a high quality such that $\psi_r = \psi_r^A$, where in equilibrium $1/2 \leq \psi_r^A \leq 1$.[22] *Figure 1* summarizes the beliefs of the two different consumer groups.
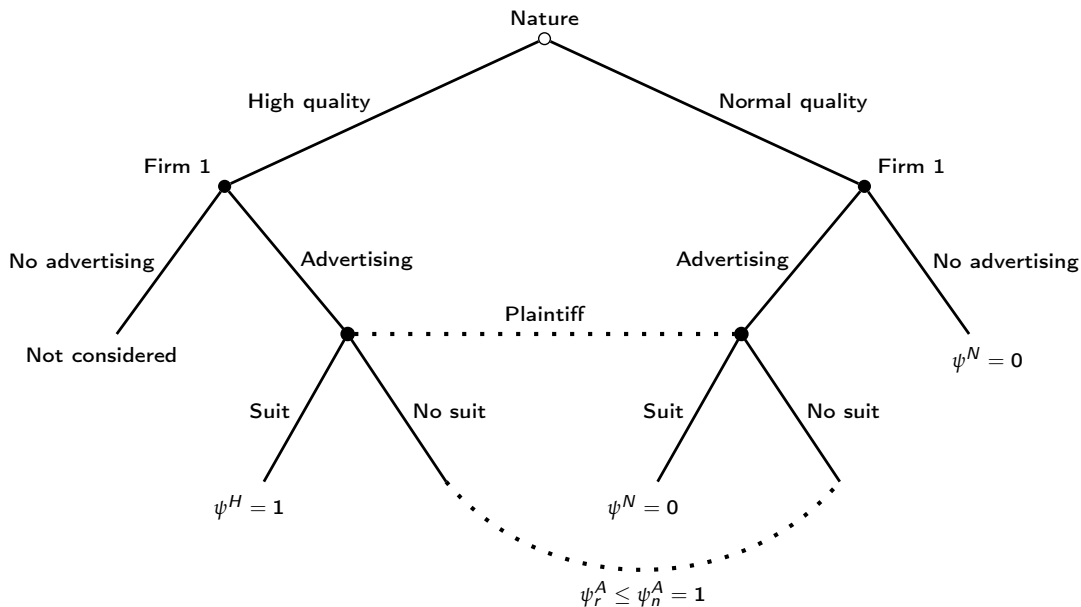

Figure 1: Consumers' beliefs.

Let us briefly comment on the motivation for analyzing different consumer groups. As far as consumer miscomprehension of advertising messages is concerned, this indeed appears to be a widespread phenomenon, which has been documented in a series of studies. Furthermore, as we assume in our model, the extent of misperception can differ among consumers. For example, Russo et al. (1981) develop a procedure to identify misleading advertising, based solely on measured consumer beliefs. They conducted an experiment in which misleadingly false product characteristics in ads were indeed perceived differently

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[22]The process of updating beliefs by rational consumers is described in more detail in the following section in which we establish the equilibrium. The lower bound on ψ_r^A follows from the assumption that firms offering high quality always advertise and that the probability of firm 1 being of the high type equals $1/2$.

In contrast to our setting, sophisticated consumers in Deng et al. (2018) are always fully informed, whereas sophisticated consumers in the present setup can face uncertainty (with respect to the product quality). Moreover, in their setup, product quality is known by all consumers, but consumer sophistication affects the (dis)utility from purchasing a product on a real (false) promotion, where the listed price serves as a reference point.

by participants within (and across) groups, i.e., groups of naïve and more sophisticated consumers could be identified.

The advertising and court decisions are followed by competition in prices. Finally, given firms' advertising, filing, and pricing decisions (possibly together with the findings in an injunction suit), consumers decide which firm to buy from and profits are realized. By assuming that the filing decision for all plaintiffs only takes place after an advertisement has been observed we follow McAfee et al. (2008) among others in that pre-commitment is possible neither for public nor private agents.[23]

For the reader's ease, we summarize the timing of the game below:

1. Firm 1 learns its quality level and decides whether or not to (truthfully or falsely) advertise high quality of its product.

2. In the event of advertising initiated by firm 1, in scenario 1 (scenario 2) firm 2 or the consumer protection agency (the government agency) can file an injunction suit focusing on false advertising claims. In case of a lawsuit, a correct ruling is rendered by the court.

3. Firms set prices for their products taking into account consumers' beliefs about product quality.

4. Consumers decide from which firm to buy.

4 Equilibrium

4.1 Price competition

We solve the model by backward induction and start by analyzing consumers' purchase decisions in the last stage.[24] Given firm 1's advertising strategy, the decision about filing an injunction suit, and the resulting belief ψ_j ($\psi_j \in \{\psi^N, \psi^H, \psi_j^A\}$ with $j = n, r$), a

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[23]Our model may be understood as combining a Hotelling (1929) duopoly with a specific kind of an inspection game. For a recent survey of using the inspection game in models of law enforcement, see Rauhut (2017). The setup can also be linked to models on frivolous litigation as in Katz (1990), where the filing of a frivolous claim is replaced with a false advertising of high product quality. What distinguishes the present setup from those contributions is our focus on plaintiffs' incentives as well as consumer naïveté.

[24]The analysis here is similar to the treatment of horizontal-plus-vertical differentiation in chapter 3.1.4 of Belleflamme & Peitz (2010).

consumer of type j who is indifferent between buying from firm 1 and firm 2 is located at $\tilde{x}_j$, where

$$v + \psi_j - p_1 - \tilde{x}_j = v - p_2 - (1 - \tilde{x}_j) \Rightarrow \tilde{x}_j = \frac{1}{2} + \frac{\psi_j - p_1 + p_2}{2}.$$

As a result, the demands realized by firm 1 and firm 2 for this consumer type amount to $\tilde{x}_j$ and $1 - \tilde{x}_j$.[25]

Given the indifferent consumer's location, firms' profit levels are given by

$$\pi_1 = p_1 \left(q\tilde{x}_n + (1 - q)\tilde{x}_r \right) = p_1 \left(\frac{1}{2} + \frac{\Psi - p_1 + p_2}{2} \right) \tag{1}$$

and

$$\pi_2 = p_2 \left(q(1 - \tilde{x}_n) + (1 - q)(1 - \tilde{x}_r) \right) = p_2 \left(\frac{1}{2} - \frac{\Psi - p_1 + p_2}{2} \right), \tag{2}$$

where $\Psi := q\psi_n + (1 - q)\psi_r$ can be interpreted as the weighted belief about firm 1's quality level.

Firms compete in prices. Starting from (1) and (2), we obtain equilibrium prices as

$$p_1 = 1 + \frac{\Psi}{3} \qquad p_2 = 1 - \frac{\Psi}{3}. \tag{3}$$

The results show that the difference in prices is increasing in the weighted belief about firm 1's quality level. Given equilibrium prices, the indifferent consumers' locations result as

$$\tilde{x}_j = \frac{1 + \psi_j}{2} - \frac{\Psi}{3}.$$

As $0 \leq \psi_r \leq \Psi \leq \psi_n \leq 1$, firm 1 always serves weakly more than half of the naïve consumers, whereas less than half of the rational consumers choose the product of firm 1 when beliefs differ starkly, and the share of naïve consumers is relatively large (which implies a relatively high price charged by firm 1). Nevertheless, for some positive belief about firm 1's quality advantage, firm 1 always serves more than half the market because of

$$D_1 := q\tilde{x}_n + (1 - q)\tilde{x}_r = \frac{1}{2} + \frac{\Psi}{6} \geq \frac{1}{2}. \tag{4}$$

To conclude the description of price competition, we report equilibrium profits which are given by

$$\pi_1 = \frac{(3 + \Psi)^2}{18} \qquad \pi_2 = \frac{(3 - \Psi)^2}{18}.$$

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[25] We implicitly assumed that both firms serve both groups of consumers, i.e., $\tilde{x}_j \in (0, 1)$, which is indeed the case in any equilibrium considered.

Profits are increasing (decreasing) in expected quality of firm 1's product for firm 1 (firm 2).

4.2 Injunction suits and advertising decision

In the following, we will analyze the private parties' (i.e., firm 2's and the consumer protection agency's) as well as the government agency's incentive to file an injunction suit (Section 4.2.2) and firm 1's incentive to falsely advertise a high product quality (Section 4.2.3). The two decisions determine the equilibrium outcomes as described in Section 4.2.4. For our analysis, we start by summarizing profits, consumer surplus, and social welfare for the possible informational situations.

4.2.1 Profits, consumer surplus, and social welfare

In order to analyze the decisions taken by the agents involved, three different cases for consumers' beliefs are relevant: beliefs in the event that (i) no advertising is observed at all, or advertising had to be stopped after an injunction had been granted, (ii) advertising can be upheld after an injunction suit, (iii) advertising takes place and no injunction suit is filed. In the following, we will report profits, consumer surplus, and social welfare (neglecting trial costs k for the moment) for the three possible cases. The results constitute the basis for the derivation of the equilibrium and the comparison of consumer and social welfare for the different possible plaintiffs.

Case (i) is associated with firm 1 being correctly identified as a normal-quality firm. Beliefs are given by $\psi_n = \psi_r = \Psi = 0$ which results in profits of

$$\pi_1^N = \pi_2^N = \pi^N = \frac{1}{2}.$$

All consumers buy a normal-quality good from the firm nearest to their location (i.e., $\tilde{x}_n = \tilde{x}_r = 1/2$) and pay an equilibrium price equal to $p_1 = p_2 = 1$. Accordingly, consumer surplus amounts to

$$CS^N = v - 1 - 2 \int_0^{\frac{1}{2}} x dx = v - \frac{5}{4}.$$

We define social welfare as the (unweighted) sum of consumer surplus and firms' profits, i.e., $SW = CS + \pi_1 + \pi_2$. Therefore, social welfare in the first case is given by

$$SW^N = v - \frac{1}{4}.$$

For case (ii), firm 1 is correctly predicted to offer high quality and $\psi_n = \psi_r = \Psi = 1$. In this case, profits are given by

$$\pi_1^H = \frac{8}{9} \qquad \pi_2^H = \frac{2}{9}.$$

For both naïve and rational consumers, the indifferent consumer is located at $\tilde{x}_j = 2/3$ (i.e., two thirds of consumers enjoy the gross valuation $v + 1$ instead of v) and prices amount to $p_1 = 4/3$ and $p_2 = 2/3$. Accordingly, we obtain consumer surplus as

$$CS^H = v + \frac{2}{3} - \frac{2}{3}\frac{4}{3} - \frac{1}{3}\frac{2}{3} - \left(\int_0^{\frac{2}{3}} x\,dx + \int_{\frac{2}{3}}^1 (1-x)\,dx \right) = v - \frac{13}{18}.$$

Social welfare amounts to

$$SW^H = v + \frac{7}{18}.$$

Finally, with advertising but no injunction suit, case (iii), we obtain $\psi_n = 1$ and $\psi_r = \psi_r^A$ (to be determined below), with corresponding profit levels equal to

$$\pi_1^A\left(\psi_r^A\right) = \frac{\left(3 + \Psi^A\right)^2}{18} \qquad \pi_2^A\left(\psi_r^A\right) = \frac{\left(3 - \Psi^A\right)^2}{18},$$

where $\Psi^A := q + (1-q)\psi_r^A$. Intuitively, profits of firm 1 (firm 2) increase (decrease) in the share of naïve consumers q and rational consumers' belief ψ_r^A.

The real quality of firm 1's product can be either high or normal in this case, where in equilibrium the belief ψ_r^A of rational consumers must coincide with the true probability for high quality. We obtain $\tilde{x}_n = 1 - \Psi^A/3$ and $\tilde{x}_r = 1/2(1 + \psi_r^A) - \Psi^A/3$ such that demand served by firm 1 amounts to $1/2 + \Psi^A/6$. Prices equal $p_1 = 1 + \Psi^A/3$ and $p_2 = 1 - \Psi^A/3$. Accordingly, the expected consumer surplus amounts to

$$\begin{aligned}
CS^A(\psi_r^A) = {} & v + \psi_r^A\left(\frac{1}{2} + \frac{\Psi^A}{6}\right) - \left(1 + \frac{\Psi^A}{3}\right)\left(\frac{1}{2} + \frac{\Psi^A}{6}\right) - \left(1 - \frac{\Psi^A}{3}\right)\left(\frac{1}{2} - \frac{\Psi^A}{6}\right) \\
& - \left(q\left(\int_0^{\tilde{x}_n} x\,dx + \int_{\tilde{x}_n}^1 (1-x)\,dx \right) + (1-q)\left(\int_0^{\tilde{x}_r} x\,dx + \int_{\tilde{x}_r}^1 (1-x)\,dx \right) \right) \\
= {} & v - \frac{5}{4} + \frac{\psi_r^A(18 + \psi_r^A) - q(1 - \psi_r^A)(9 - 5\psi_r^A) + 4q^2(1 - \psi_r^A)^2}{36} \qquad (5)
\end{aligned}$$

Expected social welfare as the sum of consumer and producer surplus amounts to

$$SW^A\left(\psi_r^A\right) = v - \frac{1}{4} + \frac{\psi_r^A}{2} + \frac{5(\psi_r^A)^2 - q(1 - \psi_r^A)(9 - 13\psi_r^A) + 8q^2(1 - \psi_r^A)^2}{36}.$$

With regard to beliefs ψ_r^A, both consumer surplus and social welfare necessarily increase in ψ_r^A which results from the direct effect of a higher probability of obtaining a

high-quality product. Interestingly, whereas aggregate consumer surplus in this case decreases in the share of naïve consumers, social welfare increases in the share of naïve consumers for a sufficiently high probability of firm 1 serving high quality ψ_r^A or a sufficiently large share of naïve consumers q.

Consumer surplus decreases with an increase in the share of naïve consumers for two main reasons: First, due to the misperception with respect to expected quality, consumer surplus for the group of naïve consumers is lower than for the group of rational consumers such that a shift towards the group of naïve consumers lowers aggregate consumer welfare. Second, the ensuing price adjustments by firms (increase in price for firm 1 and decrease in price for firm 2, see expressions (3)) lead to an increase in the average price $p_1 D_1 + p_2(1 - D_1)$ due to $D_1 > 1/2$ (see expression (4)).

As the observation with regard to the non-monotone effect of consumer naïveté on social welfare helps us explain our results below, we will have a closer look at the intuition behind it. Note that for a given probability of high quality, the market equilibrium exhibits a fundamental distortion which can be attributed to two sources. The first source is due to the fact that firms charge different prices: As firm 1 is the firm with the higher (expected) quality, i.e., there is vertical differentiation among firms, it will charge a higher price than firm 2. As a consequence, the price difference affects consumers' purchase decisions and hence firms' market shares. From a social welfare perspective, however, market shares should not depend on prices but solely on a comparison of additional transport costs and the additional benefits from consuming the product with the higher (expected) quality.[26] This means that the socially optimal consumption decision should be characterized by the type-independent indifferent consumer located at $\tilde{x}_{so}$:

$$v + \psi_r^A - \tilde{x}_{so} = v - (1 - \tilde{x}_{so}) \Leftrightarrow \tilde{x}_{so} = \frac{1 + \psi_r^A}{2}.$$

However, given firms' above-mentioned pricing decisions, there is a (standard) distortion which results in too low demand by rational consumers:

$$\tilde{x}_r = \frac{1 + \psi_r^A}{2} - \frac{\Psi^A}{3} < \tilde{x}_{so}.$$

The second source of distortion results from naïve consumers' misperception of expected quality offered by firm 1. As they overestimate the quality of product 1 but are

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[26]See, e.g., chapter 3.1.4 of Belleflamme & Peitz (2010).

also affected by the price difference between firms, the overall effect can go either way resulting in naïve consumers' over- or underconsumption of the product offered by firm 1:

$$\tilde{x}_n = 1 - \frac{\Psi^A}{3} \lesseqgtr \tilde{x}_{so} \Leftrightarrow \frac{3 - 5\psi_r^A - 2q\left(1 - \psi_r^A\right)}{6} \lesseqgtr 0.$$

From the comparison with the socially optimal level of demand by naïve consumers, it holds that there is overconsumption of the product offered by firm 1 when the share of naïve consumers is low ($q < (3-5\psi_r^A)/2(1-\psi_r^A)$), or when the probability that firm 1 offers a high-quality product is low ($\psi_r^A < (3 - 2q)/(5 - 2q)$); otherwise, also naïve consumers buy too few units from firm 1. The dependence on the share of naïve consumers can be explained by the price difference, which increases in the share of naïve consumers: When the share of naïve consumers is low, the effect from false beliefs outweighs the effect from the price difference resulting in overconsumption of the product offered by firm 1. Further, note that in our setup, the absolute distortion is always larger for the group of rational consumers in comparison to naïve consumers.

To understand how an increase in the share of naïve consumers impacts on social welfare, we note that there are two effects from such an increase: a direct effect and an indirect one. The direct effect is due to the change of consumer composition taking the price difference between firms as given. As the absolute distortion is always smaller for the group of naïve consumers, replacing rational by naïve consumers implies a positive impact on welfare due to the direct effect.

With regard to the indirect effect, we note that a higher share of naïve consumers results in a further increase in the price difference. This effect aggravates the distortion for rational consumers and ameliorates (aggravates) the distortion for naïve consumers in the event of initial overconsumption (underconsumption) of the product offered by firm 1. In contrast to the direct effect, in sum this effect is always negative—even if naïve consumers initially overconsume the product offered by firm 1. This is due to the fact that, as is usual for the model with linear transport costs, distortions enter social welfare quadratically, putting more weight on the more pronounced distortion for rational consumers.

Finally, the relative importance of the direct and indirect for social welfare again depends on the initial share of naïve consumers. In particular, note that the indirect effect becomes less important for a higher initial share of naïve consumers, as the indirect

effect is driven by the share of rational consumers which is already low. In consequence, for a relatively high (low) initial share of naïve consumers, an increase in this share raises (lowers) social welfare. As we will see below, a government agency's incentives to file suit can be explained by these observations.

4.2.2 Decision on filing an injunction suit

We can now investigate the decisions to file an injunction suit and compare incentives for the private parties to the decision problem of a government agency in the event that advertising is used by firm 1. When deciding on an injunction suit, the actors take rational consumers' belief ψ_r^A as given. Furthermore, with private plaintiffs as well as the government agency acting as rational decision-makers, in equilibrium ψ_r^A will coincide with their own assessment of the probability that the advertisement campaign has been initiated by a high-quality firm.

Firm 2 as the plaintiff

Observing an advertisement by firm 1, firm 2 has to decide whether or not to file an injunction suit. If successful, consumers' beliefs about firm 1's product quality are corrected downwards yielding an additional profit for firm 2 which obtains π^N instead of $\pi_2^A(\psi_r^A)$. When the injunction is dismissed, both types of consumers know that firm 1 offers a product of high quality reducing firm 2's profit to π_2^H and in addition firm 2 has to bear the trial costs k. Not filing an injunction suit results in profits equal to $\pi_2^A(\psi_r^A)$. The increase in expected profits from filing an injunction suit is given by

$$
\begin{aligned}
\Delta_{F2}\left(\psi_r^A\right) &= \psi_r^A\left(\pi_2^H - k\right) + \left(1 - \psi_r^A\right)\pi^N - \pi_2^A\left(\psi_r^A\right) \\
&= \frac{\left(1 - \psi_r^A\right)\left(6q + \psi_r^A(1-q)^2 - q^2\right)}{18} - \psi_r^A k.
\end{aligned}
\tag{6}
$$

Accordingly, (not) filing an injunction suit is optimal for firm 2 for $\Delta_{F2} > 0$ ($\Delta_{F2} < 0$), whereas for $\Delta_{F2} = 0$ firm 2 is just indifferent between its options. For $\psi_r^A = 1$, i.e., rational beliefs suggest only high-quality firms advertise, $\Delta_{F2} = -k < 0$ and no injunction suit will be filed. The more likely it is that the advertisement comes from a firm offering only normal quality (decrease in ψ_r^A), the more profitable an injunction suit gets in expectation, as

$$
\frac{\partial \Delta_{F2}}{\partial \psi_r^A} = \frac{1 - 2q(4-q) - 2\psi_r^A(1-q)^2}{18} - k < \left.\frac{\partial \Delta_{F2}}{\partial \psi_r^A}\right|_{\psi_r^A = \frac{1}{2},\, k=0} = -\frac{q(6-q)}{18} < 0.
$$

The maximal level of Δ_{F2} results for the minimum feasible level of ψ_r^A, $\psi_r^A = 1/2$, and consequently, we obtain an upper bound for trial costs $\hat{k}_{F2}$ such that for higher trial costs an injunction suit will never be filed:

$$\Delta_{F2}\left(\frac{1}{2}\right) = 0 \quad \Rightarrow \quad \hat{k}_{F2} := \frac{1 + q(10 - q)}{36}.$$

Further note that filing an injunction suit becomes more profitable for firm 2 in expected terms the higher the share of naïve consumers:

$$\frac{\partial \Delta_{F2}}{\partial q} = \frac{\left(1 - \psi_r^A\right)\left(3 - \Psi^A\right)}{9} > 0.$$

The higher the share of naïve consumers, the higher is the additional profit from correcting rational consumers' expectations downwards in the event of a successful lawsuit and the smaller is the unfavorable shift in demand if an injunction is not granted by the court.

Consumer protection agency as the plaintiff

We now consider a consumer protection agency as an alternative private plaintiff. We assume that the consumer protection agency aims at maximizing consumer surplus minus own trial costs.[27]

The expected gain from filing an injunction suit amounts to

$$\Delta_{CS}(\psi_r^A) = \psi_r^A\left(CS^H - k\right) + \left(1 - \psi_r^A\right)CS^N - CS^A(\psi_r^A)$$
$$= \frac{\left(1 - \psi_r^A\right)\left(9q + \psi_r^A(1 - 5q) - 4q^2\left(1 - \psi_r^A\right)\right)}{36} - \psi_r^A k,$$

Just like in the case in which firm 2 acts as the plaintiff, the expected gain from filing a suit decreases with the probability that the advertising is true, i.e.,

$$\frac{\partial \Delta_{CS}(\psi_r^A)}{\partial \psi_r^A} = \frac{-10q\left(1 - \psi_r^A\right) - 4q\left(1 - 2q\left(1 - \psi_r^A\right)\right) + 1 - 2\psi_r^A}{36} - k < 0.$$

Accordingly, this implies that a suit will never be filed for $k \geq \hat{k}_{CS}$, where

$$\Delta_{CS}\left(\frac{1}{2}\right) = 0 \quad \Rightarrow \quad \hat{k}_{CS} := \frac{1 + q(13 - 4q)}{72}.$$

---

[27]Basically, this objective function may also be associated with a government agency interested only in consumer surplus (consumer standard), which indicates the possibility of a fuzzy distinction between private and public plaintiffs in this case.

Note that the critical value for trial costs increases in the share of naïve consumers (since $q \leq 1$) as does the expected gain from an injunction suit

$$\frac{\partial \Delta_{CS}}{\partial q} = \frac{1 - \psi_r^A}{36} \left((9 - 8q) \left(1 - \psi_r^A \right) + 4 \psi_r^A \right) > 0,$$

which shows that as for firm 2, incentives to file suit increase in consumer naïveté.

Government agency as the plaintiff

Turning to the government agency, the expected increase in social welfare from an injunction suit amounts to

$$\Delta_{SW}(\psi_r^A) = \psi_r^A SW^H + \left(1 - \psi_r^A \right) SW^N - k - SW^A(\psi_r^A)$$
$$= \frac{\left(1 - \psi_r^A \right) \left(5\psi_r^A + q \left(9 - 13\psi_r^A \right) - 8q^2 \left(1 - \psi_r^A \right) \right)}{36} - k.$$

Note that, in contrast to the private plaintiffs, the government agency takes into account trial costs k as social costs irrespective of the outcome of a trial but recognizes that also the verification of high quality might raise social welfare. As for the private plaintiffs, in the event of $\psi_r^A = 1$, i.e., absent false advertising, the expected gain from an injunction suit is necessarily negative ($\Delta_{SW} = -k < 0$), and no injunction suit will be filed. Moreover, given $\psi_r^A \geq 1/2$, it also holds for the government agency that an injunction suit becomes more favorable the lower beliefs ψ_r^A are:[28]

$$\frac{\partial \Delta_{SW}}{\partial \psi_r^A} = -\frac{(2\psi_r^A - 1)(5 - 13q + 8q^2) + q(9 - 8q)}{36} < 0.$$

With ψ_r^A being constrained from below by 1/2, any increase in ψ_r^A implies relatively less uncertainty about the product's true quality and a lower value of an injunction suit form a welfare perspective. As for the private plaintiffs, this implies that we find a critical cost level $\hat{k}_{SW}$ such that an injunction suit will never be filed by the government agency for higher trial costs for any theoretically possible belief:

$$\Delta_{SW}\left(\frac{1}{2} \right) = 0 \quad \Rightarrow \quad \hat{k}_{SW} := \frac{5 + q \left(5 - 8q \right)}{144}.$$

Finally, an increase in the share of naïve consumers changes the increase in expected welfare according to

$$\frac{\partial \Delta_{SW}}{\partial q} = \frac{\left(1 - \psi_r^A \right) \left(9 - 13\psi_r^A - 16q \left(1 - \psi_r^A \right) \right)}{36}$$

[28]Note that even for $5 - 13q + 8q^2 < 0$ the numerator cannot be lower than $5 - 4q$.

which is larger than zero for low values of q and ψ_r^A but negative otherwise. That is, for a given belief ψ_r^A, an increase in the share of naïve consumers increases incentives for filing an injunction suit for private plaintiffs but can decrease incentives for the government agency. This is due to the fact described before, that in the absence of an injunction suit, social welfare can increase in the share of naïve consumers as the degree of distortions is attenuated.

4.2.3 Comparison of filing decisions

We are now in a position to compare incentives for filing an injunction suit for private plaintiffs and a government agency. Let us start by briefly summarizing the differences in the three parties' considerations. As far as trial costs are concerned, their (ex-post) relevance depends on the court decision for the private plaintiffs, as they only have to pay them in case they lose the case. In contrast, trial costs are relevant for the government agency independent of the outcome in court, because they constitute social costs. With regard to the court decision, it is true that the competitor is only interested in winning the case, since his profits are even lower if the court affirms the rival's high quality. In contrast, the dissemination of information by a court ruling may increase consumer surplus even if the consumer protection agency loses in court. The increase in consumer surplus is always obtained when winning the case, as the distortion in purchase decisions is avoided and average prices are lower. If the court confirms high quality, consumer surplus might still increase, as also rational consumers now (correctly) believe that firm 1 offers the high-quality product with certainty (no distortion). Nevertheless, the resulting price adjustments can lead to lower overall consumer welfare. Note, however that, neglecting trial costs, in expected terms consumer surplus always increases with information dissemination. Indeed, for the government agency which also considers profits, neglecting trial costs, the dissemination of information always has a positive effect even when the court confirms high quality, i.e., when the case is lost.

Firm 2 vs. consumer protection agency

We start with a comparison for the two private plaintiffs. For both private parties, trial costs are only relevant when losing the case such that any difference in incentives with regard to demand for an injunction must be due to differences in the expected valuation

of a court verdict. As described above, firm 2 only benefits from winning the case, while profits become even lower should the court affirm the competitor's quality claim. The consumer protection agency benefits from winning the case by preventing distorted purchase decisions, although consumers who stay with firm 2 pay a higher price. When losing the case, in aggregate consumers can still profit from the accompanying information revelation due to better informed purchase decisions, although consumers already buying from firm 1 suffer from an increase in price. The effects of a court verdict for the consumer protection agency are generally more mixed with some consumers benefiting, and some consumers losing in the event of either verdict compared to the evaluation by firm 2. As established before, for both private plaintiffs, the expected gain from a judgment increases in the share of naïve consumers due to the larger gain from revealing false advertisements combined with the less pronounced repercussions of losing in court. Taking together all effects, in our setting, it turns out that the expected gain from an injunction suit is always larger for firm 2 compared to the consumer protection agency, which implies that firm 2 will have greater incentives to file suit:

$$\Delta_{F2}(\psi_r^A) - \Delta_{CS}(\psi_r^A) = \frac{1 - \psi_r^A}{36} \left[3q + \psi_r^A(1 + q) + 2q^2(1 - \psi_r^A) \right] > 0, \qquad (7)$$

which is mirrored in a higher critical level for trial costs for firm 2,

$$\hat{k}_{F2} - \hat{k}_{CS} = \frac{1 + q(7 + 2q)}{72} > 0. \qquad (8)$$

Comparing the two private plaintiffs we have that either both or none of them has an incentive to sue (for $\Delta_{F2}(\psi_r^A) > \Delta_{CS}(\psi_r^A) > 0$ or $0 > \Delta_{F2}(\psi_r^A) > \Delta_{CS}(\psi_r^A)$) or it is only firm 2 that has an incentive to file an injunction suit (for $\Delta_{F2}(\psi_r^A) > 0 > \Delta_{CS}(\psi_r^A)$). We can thus summarize:

Lemma 1 *Firm 2 always has a higher expected gain from filing an injunction suit compared to the consumer protection agency, i.e., it has greater incentives to file an injunction suit.*

Private plaintiffs vs. government agency

We will focus on the comparison of the incentives of firm 2 and the government agency, as a comparison of the consumer protection agency's incentives and those of the government

agency are qualitatively similar. In order to compare the filing decisions, we calculate the difference in the expected gains from an injunction suit

$$\Delta_\Delta := \Delta_{F2} - \Delta_{SW}$$
$$= (1 - \psi_r^A)\frac{q(1 + 2q(1 - \psi_r^A) + 3\psi_r^A) - \psi_r^A}{12} + (1 - \psi_r^A)k \qquad (9)$$

which can be smaller or larger than zero. The last term indicates the different perception of trial costs, whereas the first term, which can be larger or smaller than zero, summarizes the differences in the value of information transmission by a verdict. As the expected change in welfare comprises the expected change in firm 2's profits, term 1 in expression (9) measures the sum of the expected changes in the consumer surplus and firm 1's profit. As described before, consumers on aggregate benefit from information transmission in expected terms. Firm 1 benefits if the injunction demand is dismissed but forgoes profits if its deceitful advertising is revealed. In summary, the gain from filing an injunction suit may be higher or lower for firm 2 in comparison to the governmental agency depending on the parameter values.[29] In particular, for a given belief ψ_r^A we have

$$\frac{\partial \Delta_\Delta}{\partial q} = \frac{(1 - \psi_r^A)(1 + 4q(1 - \psi_r^A) + 3\psi_r^A)}{12} > 0,$$

that is, an increase in the share of naïve consumers makes an injunction suit more favorable for firm 2 in comparison to the government agency. Indeed, for only naïve consumers, $q = 1$,

$$\Delta_\Delta|_{q=1} = (1 - \psi_r^A)\left(\frac{1}{4} + k\right) > 0,$$

which means that with only naïve consumers present the gain from an injunction suit is always higher for firm 2 than for the government agency. Note that in this case, the informational value of a verdict can never be negative for firm 2 and its gain from revealing false advertising is larger than the corresponding increase in social welfare. Instead, for only rational consumers, $q = 0$, we obtain

$$\Delta_\Delta|_{q=0} = -(1 - \psi_r^A)\left(\frac{1}{12} - k\right)$$

which is smaller zero for $k < 1/12$ implying higher expected gains from an injunction suit for the government agency for low values of trial costs.[30] In this case, the expected change

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[29] Different expected gains do not necessarily result in different actions, as the decision to file an injunction suit is a binary decision. However, differences in expected gains are necessary to induce different filing decisions.

[30] Note that for $q = 0$, we have $\hat{k}_{F2} = 1/36 < \hat{k}_{SW} = 5/144 < 1/12$ such that $k < 1/12$ is necessary for any injunctions to be filed.

in social welfare from an injunction suit is larger than the gain in profits for firm 2, and this difference can even outweigh the fact that the government agency always takes into account the full trial costs.

We may also compare the critical cost levels $\hat{k}_{F2}$ and $\hat{k}_{SW}$. The difference amounts to

$$\Delta_k := \hat{k}_{F2} - \hat{k}_{SW} = \frac{-1 + q(35 + 4q)}{144}.$$

In line with the considerations above, for the extremes of $q = 1$, only naïve consumers, and $q = 0$, only rational consumers, we find

$$\Delta_k|_{q=1} = \frac{19}{72} > 0$$

and

$$\Delta_k|_{q=0} = -\frac{1}{144} < 0.$$

Accordingly, with only naïve consumers, there is a range for trial costs k for which the government agency would never file an injunction suit, whereas firm 2 might do so. In contrast, with only naïve consumers, it might be the case that firm 2 will never file a lawsuit for some values of k for which the government agency might nevertheless go to court. Finally, since

$$\frac{\partial \Delta_k}{\partial q} = \frac{35 + 8q}{144} > 0,$$

there exists exactly one value for the share of naïve consumers q for which the two critical values for trial costs coincide.

Given qualitatively similar results with regard to the consumer protection agency's incentives, we can summarize our results so far as follows:[31]

Lemma 2 *For given beliefs about firm 1's product quality in the event of advertising, the expected gain from filing an injunction suit can be higher for a private plaintiff or the government agency. The difference in expected gains increases in favor of the private plaintiffs for a higher share of naïve consumers. This is mirrored in a higher (smaller) critical level for trial costs for the private plaintiffs in comparison to the social planner for a large (small) share of naïve consumers.*

---

[31]With firm 2 having greater incentives to file suit, the range of parameter constellations for which the consumer protection agency but not the government agency will file suit is smaller in comparison to the finding for the comparison of firm 1 and the government agency. Hence, there are more parameter constellations for which only the government agency but not the consumer protection agency will file suit. Nevertheless, the government agency (consumer protection agency) has again greater incentives to file suit for a relatively low (high) share of naïve consumers.

4.2.4 Advertising decision

Firm 1 always advertises when it is of the high type.[32] For a firm of normal type, no advertising assures a profit level of π^N. With advertising profits increase to $\pi_1^A(\psi_r^A)$ as long as no injunction suit is filed, whereas if an injunction suit is filed, firm 1's profits equal π^N again, but it has to bear the trial costs k. In consequence, the expected gain from advertising for a firm 1 offering normal quality is given by

$$\Delta_{F1} := (1-\beta)\left(\pi_1^A\left(\psi_r^A\right) - \pi^N\right) - \beta k = (1-\beta)\frac{\Psi^A\left(2+\Psi^A\right)}{18} - \beta k, \qquad (10)$$

where β is the probability of being taken to court. The firm chooses to advertise (not to advertise) for sure if the difference Δ_{F1} is larger (smaller) than zero and is indifferent between its two options for $\Delta_{F1} = 0$. Intuitively, expected profits from advertising decrease with the probability of an injunction suit and increase in the belief ψ_r^A and the share of naïve consumers (for $\beta < 1$).

4.2.5 Equilibrium

For every potential plaintiff, the equilibrium of the game can take on two forms: either an equilibrium in mixed strategies or a pure-strategy equilibrium in which an injunction suit is never filed, and therefore firm 1 always advertises.[33] In any equilibrium, the equilibrium value of ψ_r^A is determined by the advertising strategy of firm 1. Denoting by α the probability that a firm 1 with normal quality chooses to advertise we obtain

$$\psi_r^A = \frac{1}{1+\alpha} \iff \alpha = \frac{1-\psi_r^A}{\psi_r^A}.$$

Hence, in equilibrium there is an inverse relationship between consumers' beliefs about quality and firm 1's advertising decision: A higher likelihood of advertising by a normal-quality firm results in a lower belief that firm 1 offers a high-quality product. In the following, we will focus on the equilibrium value of ψ_r^A instead of α.

For a mixed-strategy equilibrium, $\psi_r^A \in (1/2, 1)$ must hold, and the equilibrium is derived from $\Delta_{F1} = 0$ and either Δ_{F2}, $\Delta_{CS} = 0$, or $\Delta_{SW} = 0$ depending on the plaintiff

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[32]Given our assumptions regarding beliefs, this is indeed a dominant strategy for a firm offering high quality.

[33]In contrast, a pure-strategy equilibrium in which a normal-quality firm 1 never advertises, or in which the plaintiff always files suit cannot exist. In the absence of false advertising the plaintiff would never file suit to which misleading advertising is a best response resulting in a contradiction. Likewise, if the plaintiff always files suit, no misleading advertisement will be observed to which filing no suits is a best response.

considered. If $\Delta_{F2} < 0$ (or $\Delta_{CS} < 0$ or $\Delta_{SW} < 0$) for every value of $\psi_r^A \in (1/2, 1)$, the corner solution of $\beta = 0$ and $\psi_r^A = 1/2$ results, which is the possible pure-strategy equilibrium. The pure-strategy equilibrium results for firm 2, the consumer protection agency, and the government agency as the plaintiff if $k > \hat{k}_{F2}$, $k > \hat{k}_{CS}$, and $k > \hat{k}_{SW}$, respectively.

For firm 2 as the potential plaintiff, from $\Delta_{F2} = 0$, a mixed-strategy equilibrium requires the belief ψ_r^A to equal

$$\psi_{F2} = \frac{1 - 2q(4 - q) - 18k + \sqrt{(1 + 4q)^2 - 36k(1 - 2q(4 - q)) + 324k^2}}{2(1 - q)^2},$$

whereas otherwise we obtain a pure-strategy equilibrium if $\psi_{F2} < 1/2$ (note that $\partial\Delta_{F2}/\partial\psi_r^A < 0$). That is, in equilibrium

$$\psi_r^A = \max\left\{\psi_{F2}, \frac{1}{2}\right\}$$

if it is firm 2's decision whether or not to file an injunction suit.[34]

A similar threshold can be defined for the case with a consumer protection agency as potential plaintiff:

$$\psi_{CS} = \frac{1 - 14q + 8q^2 - 36k + \sqrt{1296k^2 + (1 + 4q^2) - 72k(1 - 14q + 8q^2)}}{2(1 - q)(1 - 4q)}.$$

Hence, in equilibrium

$$\psi_r^A = \max\left\{\psi_{CS}, \frac{1}{2}\right\}.$$

In the scenario in which the social planner acts as the plaintiff, we solve for $\Delta_{SW} = 0$ and obtain

$$\psi_{SW} = \frac{5 - 2q(11 - 8q) + \sqrt{(5 - 4q)^2 - 144k(1 - q)(5 - 8q)}}{2(1 - q)(5 - 8q)}$$

Analogously to the private plaintiffs, in this scenario, since $\partial\Delta_{SW}/\partial\psi_r^A < 0$, the equilibrium is characterized by[35]

$$\psi_r^A = \max\left\{\psi_{SW}, \frac{1}{2}\right\}.$$

For the equilibrium probability of an injunction suit, we first state that no injunction suits are filed in a pure-strategy equilibrium in which $\psi_r^A = 1/2$ results, i.e., $\beta = 0$ in

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[34]For the extreme value of $q = 1$, we obtain $\psi_r^A = \max\{5/(5 + 18k), 1/2\}$.

[35]For the extreme value of $q = 1$, we obtain $\psi_r^A = \max\{1 - 36k, 1/2\}$.

this case. Otherwise, in a mixed-strategy equilibrium, we have to plug in the equilibrium belief ψ_r^A into expression (10) and solve $\Delta_{F1} = 0$ for β to arrive at

$$\beta\left(\psi_r^A\right) = 1 - \frac{k}{\pi_1^A\left(\psi_r^A\right) - \pi^N + k}$$
$$= 1 - \frac{18k}{18k + (3 + \psi_r^A + q(1 - \psi_r^A)^2) - 9}$$

Note that $\partial\pi_1^A/\partial\psi_r^A > 0$ and accordingly $\partial\beta/\partial\psi_r^A > 0$, i.e., the equilibrium filing probability increases in the equilibrium belief ψ_r^A, which is intuitive given that a higher belief ψ_r^A increases the profit from undetected false advertising. In consequence, false advertising becomes more attractive which, in a mixed-strategy equilibrium, must be compensated by a higher probability for an injunction suit, making false advertising less attractive.

The following lemma summarizes the equilibrium:

Lemma 3 *The equilibrium of the advertising-injunction suit game is either in pure strategies with no injunction suits and all types of firm 1 advertising or in mixed strategies:*

(i) *For firm 2 as potential plaintiff, the equilibrium is given by $\psi_r^A = 1/2$ and $\beta = 0$ for $k \geq \hat{k}_{F2}$, and $\psi_r^A = \psi_{F2}$ and $\beta = \beta(\psi_{F2})$ for $k < \hat{k}_{F2}$.*

(ii) *For the consumer protection agency as potential plaintiff, the equilibrium is given by $\psi_r^A = 1/2$ and $\beta = 0$ for $k \geq \hat{k}_{CS}$, and $\psi_r^A = \psi_{CS}$ and $\beta = \beta(\psi_{CS})$ for $k < \hat{k}_{CS}$.*

(iii) *In the scenario with the government agency as potential plaintiff, the equilibrium is given by $\psi_r^A = 1/2$ and $\beta = 0$ for $k \geq \hat{k}_{SW}$, and $\psi_r^A = \psi_{SW}$ and $\beta = \beta(\psi_{SW})$ for $k < \hat{k}_{SW}$.*

With respect to comparative statics in a mixed-strategy equilibrium, we can establish for the private plaintiffs that

$$\frac{\partial\psi_{F2}}{\partial q} > 0, \frac{\partial\psi_{CS}}{\partial q} > 0, \frac{\partial\psi_{F2}}{\partial k} < 0, \frac{\partial\psi_{CS}}{\partial k} < 0.$$

The intuition runs as follows: As pointed out before, a higher share of naïve consumers (higher trial costs) increases (decreases) the expected payoffs from an injunction suit for the private plaintiff, giving the plaintiff greater (less) incentives to file suit aggressive with regard to filing. In equilibrium this must be compensated by a decrease (an increase) in the probability of a misleading advertisement until the private plaintiff is again indifferent

between options. Due to $\partial\beta/\partial\psi_r^A > 0$, an increase in the share of naïve consumers will be accompanied by a higher probability of an injunction suit given advertising, and an increase in trial costs leads to a lower probability of an injunction suit. A higher share of naïve consumers increases the expected gain from false advertisements, which in equilibrium must be compensated by a higher probability of facing an injunction suit such that a firm 1 of normal type is indifferent again between advertising and no advertising. An increase in trial costs makes advertising less attractive, which in equilibrium will be compensated by a lower probability of facing an injunction suit.

With the government agency as potential plaintiff, we find for a mixed-strategy equilibrium $(k < \hat{k}_{SW})$

$$\frac{\partial\psi_{SW}}{\partial q} > (<)0 \text{ for } q \text{ small and } k \text{ sufficiently large (otherwise)}, \quad \frac{\partial\psi_{SW}}{\partial k} < 0.$$

Accordingly, we have that an increase in the share of naïve consumers may either lead to a lower or a higher probability of misleading advertisement. As shown before, the expected gain from an injunction suit for the government agency may be decreasing in the share of naïve consumers. In this case, the agency becomes less aggressive and in equilibrium the likelihood of misleading advertising increases (and ψ_r^A falls). Regarding trial costs, no qualitative differences arise in comparison to potential private plaintiffs.

Comparing the equilibrium outcomes for different plaintiffs, we already established that firm 2 is the more aggressive plaintiff compared to the consumer protection agency (see *Lemma 1*). In consequence we have: Either for both types of plaintiffs, a mixed-strategy equilibrium will emerge in which firm 2 files suit with a higher probability, and less advertising is observed (for $k < \hat{k}_{CS}$), or only firm 2 files suits with positive probability (for $\hat{k}_{CS} < k < \hat{k}_{F2}$).

Figure 2 summarizes the differences in the equilibrium levels when comparing one of the private plaintiffs with the public plaintiff. We start with the description of the differences in outcomes for firm 2 or the government agency as the plaintiff, depicted in *Figure 2(a)*. Given a (very) limited number of naïve consumers in the market (low value of q), less false advertising occurs when the social planner can file an injunction suit compared to the case in which the competitor can do so (light shaded areas in *Figure 2(a)*). For relatively high legal costs and a very small fraction of naïve consumers, the competitor would refrain from filing suit altogether and only the government agency may

file suit (see the upper small part of the light shaded area above the $\hat{k}_{F2}$-curve). In the lower part of the light shaded area, both private and public plaintiffs may file suit, but the government agency is more aggressive resulting in a lower equilibrium share of false advertising. In these situations, firm 2 can gain relatively little from an injunction suit, as most consumers already heavily discount any advertising claim (see also *Lemma 2*). In equilibrium, the limited incentives to file suit are anticipated by firm 1 which more often opts for false advertising.

For a higher share of naïve consumers, the opposite is true. Now firm 2 can benefit more in terms of higher profits from an injunction, as consumers are less likely to accurately discount statements about quality but simply trust the advertising message. This means that more of them go to the competitor expecting a better quality. In this case, the competitor is more *aggressive*, resulting in a lower equilibrium probability for misleading advertisements and a higher probability for an injunction suit given advertising (darker shaded area). For relatively high trial costs, it is only the competitor which may file a lawsuit (see the upper part of the dark shaded area above the $\hat{k}_{SW}$-curve), whereas in the lower part of the dark shaded area, both plaintiffs file suit with positive probability, but the likelihood that a false advertisement is initiated is lower for firm 2 as the plaintiff.

A similar reasoning holds for the comparison between the government agency and the consumer protection agency. As can be seen from *Figure 2(b)*, the qualitative findings are similar to the comparison between the government agency and firm 2 as plaintiff. However, given that the consumer protection agency is less aggressive than firm 2, the government agency is the more aggressive plaintiff for a wider range of parameter combinations.
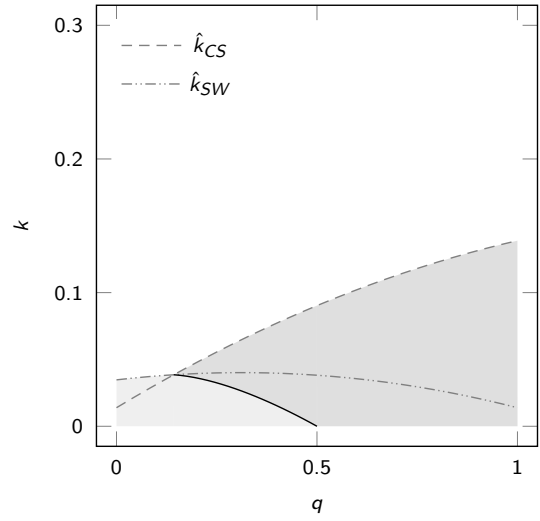
We can summarize our findings as follows:

Proposition 1 *Firm 2 is the more aggressive plaintiff than the consumer protection agency independent of the share of naïve consumers.*
The government agency is the more aggressive plaintiff than a private plaintiff for a low share of naïve consumers combined with not too high trial costs. For larger shares of naïve consumers, the private entity is the more aggressive plaintiff as long as trial costs do not negate incentives for injunction suits altogether.
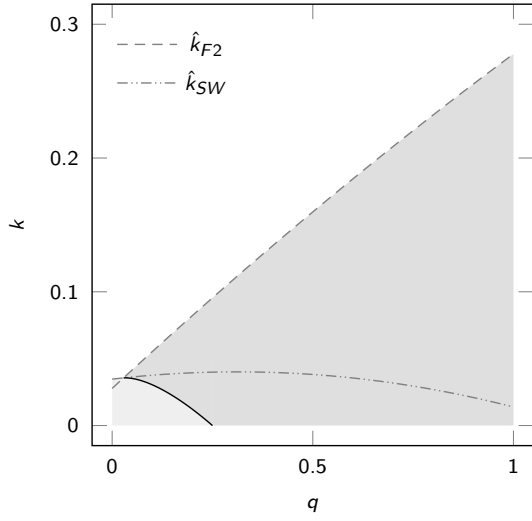
(a) Comparison with firm 2.　　　　　(b) Comparison with a consumer protection agency.

Figure 2: Comparison of rational consumers' beliefs and the scope of legal action (shaded area) under a government agency and a private plaintiff.

Note: The light (dark) shaded area represents those combinations of the share of naïve consumers and trial cost for which the government agency (firm 2 [left]/the consumer protection agency [right]) is the more aggressive plaintiff and therefore ψ_r^A is higher for the government agency (the private entity) as plaintiff.

5 Consumer surplus and social welfare

After having established the differences in the equilibrium outcomes for the possible plaintiffs, we are now in a position to provide a welfare comparison. We first investigate consumer welfare before turning to social welfare.

5.1 Consumer surplus

We start by analyzing consumer welfare (neglecting any trial costs). Four possible states can emerge, three of which involve advertising. First, firm 1 is of the high type which becomes public knowledge due to an injunction suit being filed. In this case, consumer surplus amounts to CS^H. The ex-ante probability for the state to occur is determined by the probability of firm 1 being of the high type and the equilibrium probability for an injunction suit and amounts to $\beta(\psi_r^A)/2$. Second, the injunction suit may reveal that firm 1 used misleading advertising. In this case, consumer surplus is given by CS^N. Recognizing the probability for a normal-type firm to advertise, the ex-ante probability

for this state to occur is given by $\beta(\psi_r^A)\alpha/2 = \beta(\psi_r^A)(1-\psi_r^A)/2\psi_r^A$. Third, an advertising campaign was launched, but no injunction suit has been filed. In this case, firm 1 offers high quality with probability ψ_r^A and expected consumer surplus amounts to CS^A. The ex-ante probability for this state is given by $(1-\beta(\psi_r^A))(1+\alpha)/2 = (1-\beta(\psi_r^A))/2\psi_r^A$. In addition, no advertising may be observed in the first place. In this case, firm 1 is correctly perceived to serve normal quality and consumer surplus amounts to CS^N. This state occurs with probability $(1-\alpha)/2 = (2\psi_r^A-1)/2\psi_r^A$. As long as consumers are not directly affected by trial costs, the states of detected false advertising and no advertising yield the same outcome regarding consumer welfare. In sum the ex-ante expected consumer surplus is given by

$$\mathbb{E}\left[CS\right] = \frac{\beta\left(\psi_r^A\right)}{2} \cdot CS^H + \frac{1-\beta\left(\psi_r^A\right)}{2\psi_r^A} \cdot CS^A(\psi_r^A) + \frac{2\psi_r^A - 1 + \beta(\psi_r^A)\left(1-\psi_r^A\right)}{2\psi_r^A} \cdot CS^N.$$

Regarding expected consumer welfare, it results that consumers always prefer the more aggressive plaintiff, which is associated with both less false advertising (i.e., a higher equilibrium belief ψ_r^A) and a higher filing rate β in equilibrium. In the event of observing an advertisement and given that consumer welfare does not include trial costs, first, consumers always prefer to get informed about true product quality, i.e., they prefer a higher filing rate ($\partial\mathbb{E}\left[CS\right]/\partial\beta > 0$). Second, a reduction in false advertising has a direct positive effect on consumer welfare ($\partial\mathbb{E}\left[CS\right]/\partial\psi_r^A > 0$) due to the more informative advertising. Obviously, consumers are indifferent between plaintiffs if neither of them files an injunction suit with positive probability. Hence, as long as trial costs are not borne by consumers, we find that given the commitment problem existent for all plaintiffs, consumers are better off under the competitor as potential plaintiff than under the consumer protection agency. Moreover, *Figure 2* with the distinction of the light and dark shaded areas also illustrates the comparison of expected consumer surplus for private versus public plaintiffs. We summarize:[36]

Proposition 2 *Expected consumer welfare is always higher for the more aggressive plaintiff. Accordingly, consumers prefer the government agency as plaintiff for a low share of naïve consumers combined with low trial costs. Otherwise consumers prefer the private plaintiff as long as trial costs are not too high to prevent the filing of injunction suits in*

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[36]The observation that consumers prefer the more aggressive plaintiff is true for the subgroups of both rational and naïve consumers.

equilibrium. With consumers bearing no trial costs, expected consumer welfare is always at least as high for firm 2 as the plaintiff as for the consumer protection agency as plaintiff.

5.2 Social welfare

In order to assess from a welfare perspective whether a private or public entity should be put in the position to decide on an injunction suit against firm 1, we need to evaluate the resulting expected levels of ex-ante social welfare. Following the procedure from Section 5.1 but now accounting in addition for profits and trial costs in the event of an injunction suit, expected social welfare is described by

$$\mathbb{E}\left[SW\right] = \frac{\beta\left(\psi_r^A\right)}{2} \cdot \left(SW^H - k\right) + \frac{\beta(\psi_r^A)(1 - \psi_r^A)}{2\psi_r^A} \cdot \left(SW^N - k\right)$$
$$+ \frac{1 - \beta\left(\psi_r^A\right)}{2\psi_r^A} \cdot SW^A(\psi_r^A) + \frac{2\psi_r^A - 1}{2\psi_r^A} \cdot SW^N. \tag{11}$$

Given the equilibrium outcomes in the three scenarios, we obtain the expected welfare levels $\mathbb{E}\left[SW_{F2}\right]$, $\mathbb{E}\left[SW_{CS}\right]$, and $\mathbb{E}\left[SW_{SW}\right]$.

With regard to the private plaintiffs, the resulting consequences for ex-ante expected social welfare are as follows. Again for a mixed-strategy equilibrium under both plaintiffs, welfare is higher with the more aggressive plaintiff firm 2. In the event that only firm 2 files suit with positive probability, welfare is only higher for firm 2 as potential plaintiff when the share of naïve consumers is relatively low but otherwise welfare is higher for the consumer protection agency as (inactive) plaintiff. If the share of naïve consumers is high, we already know that both private plaintiffs are more aggressive than the government agency and that private incentives can even be excessive resulting in lower ex-ante expected welfare. This is especially likely for firm 2 as the more aggressive private plaintiff which explains that with a large share of naïve consumers and only firm 2 using injunction suits, ex-ante expected welfare is higher for the consumer protection agency as plaintiff.

A comparison of the expected levels of social welfare for private and public plaintiffs yields our main result:

Proposition 3 *Expected social welfare is higher for the government agency as a plaintiff as long as the government agency is more aggressive with regard to filing an injunction suit. When the private plaintiff is the more aggressive plaintiff, we have that expected social welfare is higher for the private plaintiff when both the government agency and*

the private plaintiff will file suit with some positive probability. When only the private plaintiff will sometimes file suit, expected social welfare is higher for the private plaintiff (the government agency) for intermediate (high) trial costs.

As described in *Proposition 1*, the question of who is the more aggressive plaintiff depends to a large extent on the shares of rational and naïve consumers (in combination with the level of trial costs). *Figure 3* compares the levels of expected social welfare. As *Proposition 3* and a comparison with *Figure 2* highlight, the result is mainly driven by the plaintiff's incentives to file an injunction suit against firm 1. Indeed, the lower of the two black lines in *Figure 3* is the same as the one in the previous figure. We observe that when there is less false advertising under the government agency (higher ψ_r^A), i.e., the government agency is more aggressive with regard to filing an injunction suit, the ex-ante expected welfare is higher under a government agency as plaintiff (see the light shaded area on the left). In a mixed-strategy equilibrium in which there is less false advertising for the private plaintiff, expected social welfare is higher for the private plaintiff (lower part of the dark shaded area). When only the private plaintiff has an incentive to file injunction suits, social welfare is higher for the private plaintiff as long as trial costs are not too large (upper part of the dark shaded area); otherwise social welfare is higher for the government agency as (inactive) potential plaintiff.

The results can be explained by resorting to equation (11). Social welfare is described as a function of ψ_r^A and therefore (an inverse measure of) the extent of false advertising. In slight abuse of the model, we consider an exogenous change in the probability of high quality being offered conditional on advertising, i.e. an exogenous increase in ψ_r^A. We obtain

$$\frac{d\mathbb{E}\left[SW\right]}{d\psi_r^A} = \frac{\partial \mathbb{E}\left[SW\right]}{\partial \psi_r^A} + \frac{\partial \mathbb{E}\left[SW\right]}{\partial \beta(\psi_r^A)}\frac{\partial \beta(\psi_r^A)}{\partial \psi_r^A}.$$

The direct (partial) effect of higher informational content of advertising, i.e., an increase in ψ_r^A, is positive, as

$$\frac{\partial \mathbb{E}\left[SW\right]}{\partial \psi_r^A} = \frac{1-\beta}{72(\psi_r^A)^2}\left[q + 8q(1-q)(1-(\psi_r^A)^2) + 5(1-q)(\psi_r^A)^2\right] + \frac{\beta}{2(\psi_r^A)^2}k > 0.$$

When the government agency is more aggressive with respect to filing an injunction suit than the private plaintiff, the equilibrium with the government agency displays lower false advertisement, i.e., $\psi_{F2}, \psi_{CS} > \psi_{SW}$. Since with respect to the filing decision, the
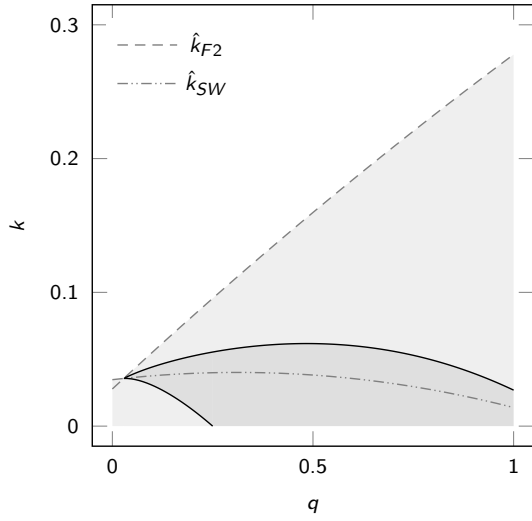
(a) Firm 2 vs. government agency.
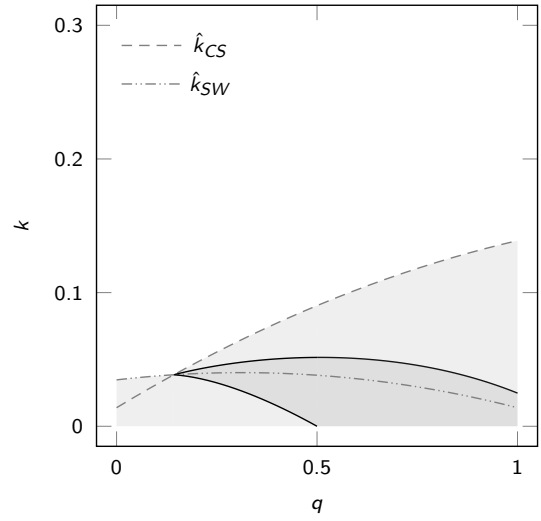
(b) Consumer protection agency vs. government agency.

Figure 3: Comparison of expected social welfare and scope of legal action under both private-plaintiff scenarios.

Note: The light (dark) shaded area represents those combinations of the share of naïve consumers and trial cost for which expected social welfare is higher under the government agency (a private plaintiff).

government agency which considers (ex-post) social welfare is just indifferent, it holds that $\partial \mathbb{E}\left[SW\right]/\partial \beta(\psi_r^A) \approx 0$ near equilibrium. Accordingly, welfare is strictly higher for the government agency as plaintiff.

When the private entity is the more aggressive plaintiff, expected social welfare can be higher or lower than for the government agency as plaintiff. As long as a mixed-strategy equilibrium results for both types of plaintiffs (lower part of the dark shaded area in *Figure 3*), expected welfare is necessarily higher for the private plaintiff. Again, it holds that $\partial \mathbb{E}\left[SW\right]/\partial \beta(\psi_r^A) \approx 0$, whereas the direct effect of a lower probability of false advertisements leads to a higher welfare level with the private plaintiff (since $\psi_{F2}, \psi_{CS} > \psi_{SW}$). Although the government agency considers social welfare when deciding about an injunction suit, it is constrained by the commitment problem described above. Ideally, it would commit to a more aggressive filing behavior in order to make false advertising less attractive. Given the government agencies commitment problem, it can therefore be beneficial from a welfare point of view to appoint the more aggressive the private entity as the potential plaintiff.

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The positive effect of a lower probability for false advertisements is still present when the government agency would abstain completely from filing injunction suits because of the high trial costs, whereas the private plaintiff still files an injunction suit with positive probability. However, in these circumstances, the direct positive effect of fewer false advertisement is counteracted by a negative effect on social welfare since now $\partial \mathbb{E}\left[SW\right]/\partial \beta(\psi_r^A) < 0$ and $\beta(\psi_{F2}), \beta(\psi_{CS}) > \beta(\psi_{SW}) = 0$. Given the extent of false advertising, the ex-post gain in social welfare from an injunction suit is more than offset by higher overall trial costs. Summing up the positive and negative effects of the more aggressive behavior of the private plaintiff in comparison to the government agency, we find that for intermediate levels of trial costs expected welfare is still higher for the private plaintiff (upper parts of the dark shaded areas in *Figure 3*), whereas the opposite results for high levels of trial costs.

6   Extensions

In this section, we discuss three extensions to our main setup. First, we present the results when the American cost sharing rule is applied. Second, we describe possible equilibria when instead of one two potential plaintiffs may file suit. Finally, we discuss the repercussions of additional fines for revealed false advertisements.

6.1   American cost sharing rule

In this section, we briefly comment on the robustness of our results with respect to the applied cost sharing rule. So far, we have considered the English rule, which stipulates that the losing party has to bear the full trial costs. However, whereas this rule applies, for instance, in many European countries, other jurisdictions rely on the American rule under which each party bears its own trial costs and no compensation takes place. In order to introduce the American rule into our model, we assume that originally total trial costs k can be ascribed in equal parts to the defendant and plaintiff, i.e., each party bears costs of $k/2$ irrespective of the trial outcome. Note that we restrict our attention to the comparison of firm 2 and the government agency—a comparison of the consumer protection agency and the government agency again gives qualitatively similar results.

Following the analysis from the main part of our paper, the allocation of trial costs to involved parties becomes only relevant when we get to the decisions on filing an injunction

suit and whether or not to use advertising. In comparison to equation (6), for firm 2 as plaintiff, the relevant increase in expected profits from an injunction suit becomes

$$\Delta_{F2}^{Am}(\psi_r^A) = \psi_r^A \pi_2^H + (1 - \psi_r^A)\pi^N - \frac{k}{2} - \pi_2^A(\psi_r^A)$$

$$= \Delta_{F2}(\psi_r^A) + \left(\psi_r^A - \frac{1}{2}\right)k.$$

Since $\psi_r^A \geq 1/2$, the expected gain from an injunction suit is larger for firm 2 under the American rule than under the English rule and, in this sense, firm 2 becomes a more aggressive plaintiff. In contrast, no change occurs for the government agency as potential plaintiff because as before the agency always takes into account full trial costs irrespective of their allocation (i.e., $\Delta_{SW}^{Am}(\psi_r^A) = \Delta_{SW}(\psi_r^A)$).[37]

In comparison to equation (10), for a firm 1 of the normal type, the expected gain from a false advertisement changes to

$$\Delta_{F1}^{Am}(\psi_r^A) = (1 - \beta)\left(\pi_1^A(\psi_r^A) - \pi^N\right) - \beta\frac{k}{2} = \Delta_{F1}(\psi_r^A) + \beta\frac{k}{2}. \tag{12}$$

Under the English rule, a firm 1 of the normal type always has to bear full trial costs in the event of a lawsuit. Accordingly, a switch to the American rule implies a reduction in expected costs and higher incentives to falsely advertise a high product quality.[38]

The equilibrium outcomes are described in *Figures 4*. The qualitative insights are the same as for the English cost allocation rule; however, some quantitative differences emerge. *Figure 4(a)* illustrates the differences in the incentives to file an injunction suit, which are then mirrored in the share of false advertising in equilibrium. For a low share of naïve consumers, only the government agency files suit with a positive probability (small upper light area), or both plaintiffs may file suit with a positive probability, but the government agency is the more aggressive plaintiff (lower light area). For medium to high shares of naïve consumers, it is firm 2 that is the more aggressive plaintiff. Still both types of plaintiff may file suit (lower dark area), or only firm 2 files suit with a positive

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[37]Note that both critical cost values $\hat{k}_{F2}$ and $\hat{k}_{SW}$ are the same as before. For the government agency, this is trivial; for firm 2 as the plaintiff, this results from the fact that in our model for $\psi_r^A = 1/2$, firm 2 incurs the same expected trial costs under both cost sharing rules.

[38]As a firm 1 of the high quality type has to bear trial costs under the American rule one might consider whether it may abstain from advertising high quality. However, this can never be the case in equilibrium. In any equilibrium, it holds that $\Delta_{F1}^{Am}(\psi_r^A) \geq 0$ for a firm 1 of normal type. Since the benefit from advertising is larger for a firm 1 of the high type in comparison to a firm 1 of the normal type, a firm 1 of the high type will always use advertising in equilibrium also for the American rule.

probability (upper dark area). The main difference to the findings under the English rule (see *Figure 2*) is that with the American rule in place, the government agency is the more aggressive plaintiff for fewer parameter combinations. This result follows directly from the above insight that the change in the cost sharing rule makes filing an injunction suit more attractive for firm 2, but at the same time leaves incentives unchanged for the government agency for given consumer beliefs.
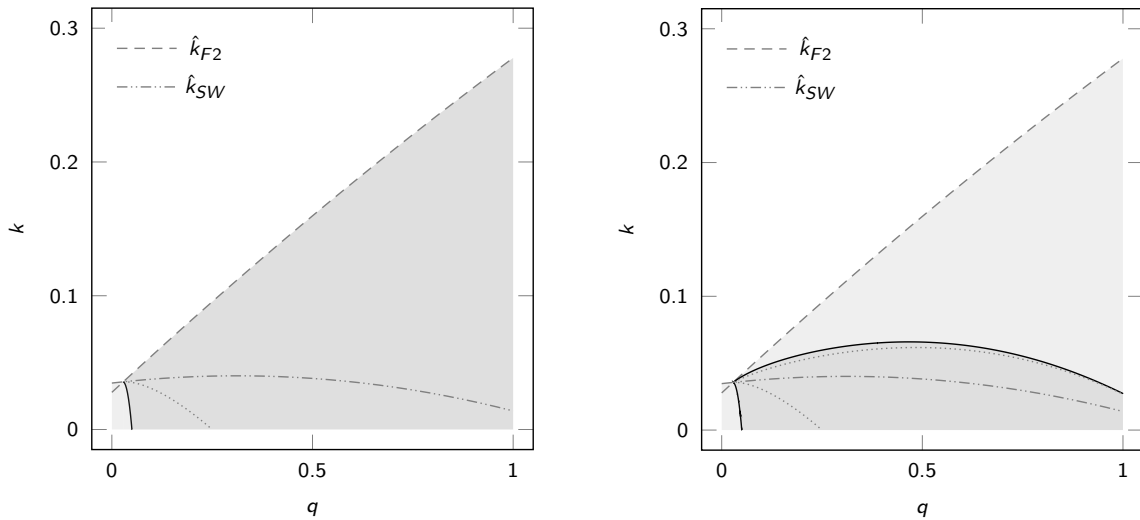
(a) Comparison of rational consumers' beliefs and the scope of legal action (shaded area).

(b) Comparison of expected social welfare and scope of legal action.

Figure 4: Comparison of rational consumers' beliefs and the scope of legal action under both plaintiff scenarios and the American rule.

Note: Left—The gray dotted line is taken from *Figure 3(a)*. The light (dark) shaded area represents those combinations of the share of naïve consumers and trial cost for which the government agency (firm 2) is the more aggressive plaintiff.
Right—The gray dotted lines are taken from *Figure 3(a)*. The light (dark) shaded area represents those combinations of the share of naïve consumers and trial cost for which expected social welfare is higher when the government agency (firm 2) acts as the plaintiff.

Figure 4(b) compares the scope of legal action and the expected social welfare for the two plaintiffs. The picture is again similar to the one found for the English rule. Social welfare is higher for the government agency as plaintiff as long as the government agency is more aggressive than firm 2. In contrast, for firm 2 being more aggressive, expected social welfare is higher for firm 2 (the government agency) as plaintiff for low to medium (high) trial costs. Again the main difference to the scenario with the English cost sharing rule in place is that firm 2 is the more aggressive plaintiff for more parameter combinations

(q, k).

6.2 More than one possible plaintiff

Assume that two instead of one agent are allowed to file an injunction suits and decide simultaneously about their demands. If one or none of the agents files an injunction suit, the proceedings are as described in the main analysis. Should both agents file suit, we assume that they share trial costs k in the event of losing the case.[39]

For firm 2, the expected gain from filing suit now amounts to

$$
\begin{aligned}
\Delta_{F2}^{new} &= \psi_r^A \left[\pi_2^H - (1 - \beta_{nF2})k - \beta_{nF2}\frac{k}{2} \right] + \left(1 - \psi_r^A\right)\pi^N \\
&\quad - \left[\beta_{nF2}\left(\psi_r^A\pi_2^H + (1 - \psi_r^A)\pi^N\right) + (1 - \beta_{nF2})\pi_2^A(\psi_r^A) \right] \\
&= (1 - \beta_{nF2})\Delta_{F2} - \psi_r^A\beta_{nF2}\frac{k}{2}
\end{aligned}
\tag{13}
$$

where β_{nF2} is the probability that the other agent has decided to file suit. The first line in equation (13) describes expected payoffs if firm 2 files suit, the second line describes expected payoffs if it refrains from doing so. As the summarized terms in the third line indicate, the effect of a second potential plaintiff on the expected benefit from filing for firm 2 are as follows: if the other agent does not file suit, which happens with probability $1 - \beta_{nF2}$, the filing by firm 2 results in the expected gain Δ_{F2} as before; if the other agent files suit, filing by firm 2 only results in bearing half of trial costs in the event that firm 1 offers high quality. Similarly, for the consumer protection agency, we obtain

$$
\Delta_{CS}^{new} = (1 - \beta_{nCS})\Delta_{CS} - \psi_r^A\beta_{nCS}\frac{k}{2},
$$

with β_{nCS} defined analogously to β_{nF2}. For the government agency, the analysis is different. We have

$$
\begin{aligned}
\Delta_{SW}^{new} &= \psi_r^A SW^H + (1 - \psi_r^A)SW^N - k \\
&\quad - \left[\beta_{nSW}(\psi_r^A SW^H + (1 - \psi_r^A)SW^N - k) + (1 - \beta_{nSW})SW^A(\psi_r^A) \right] \\
&= (1 - \beta_{nSW})\Delta_{SW}.
\end{aligned}
$$

With the government agency taking into account trial costs independent of the outcome of trial, the agent's expected gain form demanding quality verification is not affected by the shift in trial costs between possible plaintiffs.

---

[39]Alternatively, we could assume that a fair random mechanism decides who of the two parties is going to be the active plaintiff if both agents file suit. With risk-neutral parties, the analysis is the same.

The expected gain from false advertising for firm 1 is the same as in the main analysis, i.e., Δ_{F1}. With β_1 and β_2 denoting the probabilities for filing of an injunction suit by possible plaintiff 1 and possible plaintiff 2, the expected probability of facing trial, β, results as

$$\beta = \beta_1 + \beta_2 - \beta_1\beta_2. \tag{14}$$

6.2.1 Firm 2 and the government agency can file suit

With firm 2 and the government agency as possible plaintiffs, we use $\beta_1 = \beta_{nSW} = \beta_{F2}$ and $\beta_2 = \beta_{nF2} = \beta_{SW}$. Extending the discussion in the main paper, an equilibrium with a positive probability for the demand for quality verification requires $\Delta_{F1} = 0$ and either $\Delta_{F2}^{new} = 0 \ \wedge \ \Delta_{SW}^{new} \leq 0$ or $\Delta_{F2}^{new} \leq 0 \ \wedge \ \Delta_{SW}^{new} = 0$.

The equilibrium established in the main paper still constitutes an equilibrium of the extended game. With firm 2 being the more aggressive agent and trial costs low enough to allow for injunction suits in equilibrium, an equilibrium is given by $\Delta_{F2}^{new} = \Delta_{F2} = 0$, $\Delta_{SW} < \Delta_{SW}^{new} < 0$ and $\Delta_{F1}{=}0$ with $\beta_{F2} = \beta > 0$ and $\beta_{SW} = 0$. Similarly, with the government agency as the more aggressive plaintiff, a possible equilibrium (for low enough trial costs) exists in $\Delta_{F2}^{new} < 0$, $\Delta_{SW}^{new} = \Delta_{SW} = 0$ and $\Delta_{F1} = 0$ with $\beta_{F2} = 0$ and $\beta_{SW} = \beta > 0$.

However, there may exist an additional equilibrium with both possible plaintiffs demanding quality verification with positive probability. Such an equilibrium requires that all three parties, the two possible plaintiffs and a firm 1 offering normal quality, are indifferent between their possible actions, $\Delta_{F2}^{new} = \Delta_{SW}^{new} = \Delta_{F1} = 0$. We note that this is only possible if firm 1 is originally the more aggressive plaintiff, that is, $\Delta_{F2} > \Delta_{SW}$ as otherwise $\Delta_{F2}^{new} = 0$ cannot be fulfilled for $\beta_{SW} > 0$ (see equation (13)). Since the equations from the full model become quite complex without allowing for much further insight, we restrict ourselves and solve for the equilibrium for $q = 1$, i.e., when all consumers are naïve. These calculations should allow for highlighting the equilibrium properties should both possible plaintiffs file suit with positive probability.

For $q = 1$ we have

$$\Delta_{F2}^{new} = (1 - \beta_{SW})(1 - \psi_r^A)\frac{5}{18} - (2 - \beta_{SW})\psi_r^A\frac{k}{2}$$

$$\Delta_{SW}^{new} = (1 - \beta_{F2})\left[(1 - \psi_r^A)\frac{1}{36} - k\right]$$

$$\Delta_{F1} = (1 - \beta)\frac{1}{6} - \beta k$$

With equation (14), an interior solution is described by

$$\psi_r^A = 1 - 36k$$

$$\beta = \frac{1}{1 + 6k}$$

$$\beta_{F2} = 2 - \frac{20}{1 - 36k} + \frac{13}{7 + 42k}$$

$$\beta_{SW} = 2 - \frac{20}{19 + 36k}$$

This equilibrium is only feasible as long as $1/2 < \psi_r^A < 1$ and $0 < \beta_{F2}, \beta_{SW} < 1$ which requires $0 < k < \frac{\sqrt{39}-6}{36} \approx .0068 = \hat{k}_{F2,SW}^{mixed}$ and is smaller than the two critical values $\hat{k}_{F2} = 5/18 \approx .27778$ and $\hat{k}_{SW} = 1/72 = .01389$.

In summary: For very low trial costs, a second possible equilibrium exists with both firm 2 and the government agency filing suit with positive probability. In comparison to the other possible equilibrium (only firm 2 might file suit), a higher share of false advertising will result in this equilibrium (lower ψ_r^A); more precisely, in this equilibrium, the probability that the share of false advertising equals the outcome if the government agency were the only possible plaintiff (this is due to Δ_{SW}^{new} being a multiple of Δ_{SW}). That is, the existence of a second possible plaintiff may result in a kind of free-rider problem resulting in more false advertising in equilibrium. However, this requires rather low trial costs ($k < \hat{k}_{F2,SW}^{mixed}$). For $\hat{k}^{mixed} < k < \hat{k}_{F2}$ the equilibrium with only firm 2 filing suit constitutes the only equilibrium.

6.2.2 The consumer protection agency and the government agency as possible plaintiffs

Denote $\beta_1 = \beta_{nSW} = \beta_{CS}$ and $\beta_2 = \beta_{nCS} = \beta_{SW}$. The argumentation is the same as in the subsection on firm 2 and the government agency as possible plaintiffs. In the following, we concentrate on the case of a possible equilibrium with both agents demanding quality

verification with positive probability for $q = 1$. The relevant new equation is

$$\Delta_{CS}^{new} = (1 - \beta_{SW})(1 - \psi_r^A)\frac{5}{36} - (2 - \beta_{SW})\psi_r^A\frac{k}{2}$$

Now, an interior solution is described by

$$\psi_r^A = 1 - 36k$$
$$\beta = \frac{1}{1 + 6k}$$
$$\beta_{CS} = 2 - \frac{10}{7(1 - 36k)} + \frac{3}{7 + 42k}$$
$$\beta_{SW} = \frac{8 + 72k}{9 + 36k}$$

This equilibrium is only feasible as long as $1/2 < \psi_r^A < 1$ and $0 < \beta_{CS}, \beta_{SW} < 1$ which requires $0 < k < \frac{\sqrt{61}-7}{72} \approx .0113 = \hat{k}_{CS,SW}^{mixed}$. The critical value for trial costs is smaller than both the critical values $\hat{k}_{CS} = 5/36 \approx .13889$ and $\hat{k}_{SW} = 1/72 = .01389$.

In summary: For low values of trial costs, a second equilibrium exists with both plaintiffs filing suit with positive probability instead of only the consumer protection agency filing suit. In this case, the probability of false advertising coincides with the equilibrium outcome with the less aggressive governmetn agency as the only potential plaintiff.

6.2.3 Firm 2 and the consumer protection agency as possible plaintiffs

We denote $\beta_1 = \beta_{nCS} = \beta_{F2}$ and $\beta_2 = \beta_{nF2} = \beta_{CS}$. Note that firm 2 is always the more aggressive plaintiff compared to the consumer protection agency. Comparable to the discussion so far, as long as $k < \hat{k}_{F2}$ firm 2 being the only active plaintiff constitutes an equilibrium. However, a second equilibrium might exist with both firm 2 and the consumer protection agency filing suit with positive probability. For this, we must have $\Delta_{F2}^{new} = \Delta_{CS}^{new} = \Delta_{F1} = 0$. Again, we exemplify results for $q = 1$.

For $q = 1$, we obtain as a possible interior solution

$$\psi_r^A = \frac{10}{10 + 27k + 3\sqrt{3k(4 + 27k)}}$$

$$\beta = \frac{1}{1 + 6k}$$

$$\beta_{F2} = \frac{1 + 3k - \sqrt{3k(4 + 27k)}}{1 + 6k}$$

$$\beta_{CS} = \frac{2 + 15k - \sqrt{3k(4 + 27k)}}{2(1 + 6k)}$$

Given the restrictions $1/2 < \psi_r^A < 1$ and $0 < \beta_{F2}, \beta_{CS} < 1$, this equilibrium is feasible for $0 < k < 1/12 \approx .08333 = \hat{k}_{F2,CS}^{mixed}$. This compares to the critical trial costs $\hat{k}_{CS} = .13889$ and $\hat{k}_{F2} = .27778$. In summary: Again, for low values of trial costs this second equilibrium exists.

Figure 5 compares the outcome in the probability of truthful advertising ψ_r^A (i.e. an inverse measure of the probability of false advertising by firm 1) for $k < \hat{k}_{F2,CS}^{mixed}$ for either one or two active plaintiffs.
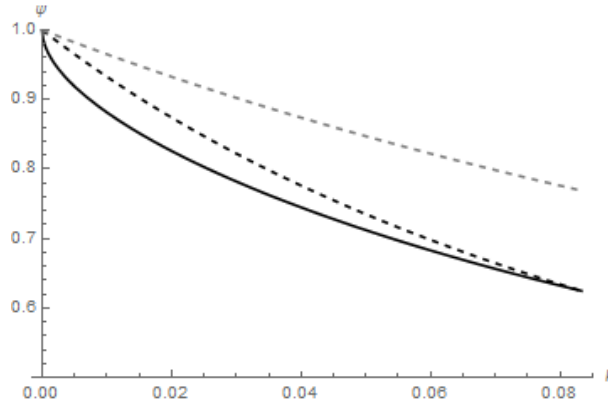

Figure 5: Equilibrium probabilities of an advertising being truthful; black, solid: firm 2 and consumer protection agency as active plaintiffs, black, dashed: only consumer protection agency as only possible plaintiff, gray, dashed: only firm 2 as active plaintiff

The solid black curve indicates the equilibrium probability that an observed advertisement is truthful if both firm 2 and the consumer protection agency file injunction suits with positive probability. The dashed black curve is relevant if only the consumer protection agency can file injunction suits. The grey dashed curve illustrates the case with only firm 2 as possible plaintiffs; the latter coincides with the other possible equilibrium

if both agents could file suit but only firm 2 does so in equilibrium. Figure 5 illustrates again that the problem of possible free-riding can result in less truthful advertising in equilibrium.

6.3 Damages and fines

In our main analysis, we only considered that, in the event of an identified false advertising, firm 1 must stop its advertising campaign and bear full trial costs. Depending on the jurisdiction considered, the firm might also have to pay damages or fines.[40] In our simplified setup, court rulings always occur before purchase decisions have been made, such that, in the case of identified false advertising, no harm occurred, rendering damages claims moot. However, with some reservation, the model may be reinterpreted in that some purchases already have taken place before verification of quality, and firm 1 has to pay compensation to harmed parties.

The inclusion of a fine payment produces additional results. Assume a monetary sanction s in the event of identified false advertising that, following the usual approach in the law and economics literature, is a pure transfer (see, e.g., Polinsky and Shavell 2007). The sanction does not directly bear on the discussion of plaintiffs' incentives to file an injunction suit, but it directly enters the calculus by a firm 1 offering normal quality over whether to advertise. Indeed, the gain from advertising described in expression (10) becomes

$$\Delta_{F1}^s := (1 - \beta)\left(\pi_1^A(\psi_r^A) - \pi^N\right) - \beta(k + s) = \Delta_{F1} - \beta s, \tag{15}$$

such that in any equilibrium with a positive probability for injunction suits, we have

$$\beta = \frac{\pi_1^A(\psi_r^A) - \pi_1^N}{\pi_1^A(\psi_r^A) - \pi_1^N - (k + s)}. \tag{16}$$

At the same time, the equilibrium probability of false advertising is not affected by the sanction. In a mixed-strategy equilibrium, this is because the probability results from the requirement of making possible plaintiffs (whose objective function has not changed) indifferent; in a pure-strategy equilibrium, all firms use advertising. In summary, the existence of the fine leaves the frequency of false advertising unaffected but reduces the

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[40]For example, in *POM Wonderful LLC v. Coca Cola Co.*, the plaintiff was asking 77 million US$ for lost business. Likewise, the FTC may impose fines in the case of false advertising. In Germany, the *Act Against Unfair Competition (UWG)* allows for competitors' claims on compensation for damages (UWG section 9) and the confiscation of profits in the event of harm to numerous purchasers (UWG section 10).

rate of injunction suits in a mixed-strategy equilibrium. In consequence, the higher the sanction the lower is the equilibrium probability that false advertising is identified, and the more likely is the outcome with advertising but no verification.

Whereas the ranking regarding plaintiffs' aggressiveness does not change, the fact that there is less demand for injunction suits may bear on welfare. With the frequency of false advertising being unaffected by the sanction, welfare effects only emerge via the ex post trade-off between trial costs and the ex post gain in welfare from revealing quality. For the government agency, this effect is necessarily zero, as in this case, any equilibrium with injunction suits is already characterized by socially efficient ex post decisions. In contrast, in the event of a private agent, welfare increases with the sanction in the range in which the agent is more aggressive than the government agency, and welfare decreases in the range in which it is less aggressive. This mirrors the ex post excessive incentives to file injunction suits in the case in which the private plaintiff is more aggressive than the government agency, and the insufficient incentives in the case in which the private plaintiff is less aggressive than the government agency. Accordingly, welfare results described in *Proposition 3* remain qualitatively unaffected. However, the range of parameter constellations in which only the private agent files suit and welfare is higher for this agent is larger due to the saving of costs as a consequence of the lower equilibrium rate of injunction suits.[41]

7 Conclusion

False advertising is a widespread phenomenon. Interestingly, jurisdictions across the world use quite different policies for law enforcement to fight false advertising campaigns and to protect consumers. In this paper, we use a duopoly model to analyze incentives to make false advertising claims about product quality. In contrast to much of the literature, we use a specific design to integrate the mode of law enforcement. Whereas in many models, an expected fine is incorporated and associated with false advertising, we explicitly use an inspection game to model incentives for filing an injunction suit against false advertis-

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[41]Note that in our setting, the court always rules correctly on false advertising. In the law and economics literature, it is a standard result that in this case, monetary sanctions should be used to their maximum, as it allows for lower enforcement costs without jeopardizing deterrence (see, e.g., Polinsky and Shavell 2007). The use of less-than-maximal fines may, however, be advisable in the event of possible court errors (see, for example, Andreoni 1991 and for a setting with false advertising, see Corts 2014a).

ing and—in line with observations from various jurisdictions—distinguish between three different potential plaintiffs: a government agency interested in social welfare, the advertising firm's competitor in the duopoly market, and a consumer protection agency. With regard to consumers, we allow for two different kinds of consumers: rational consumers, who discount unverified advertising claims according to Bayesian updating, and naïve consumers, who fully trust unverified advertising statements.

As far as the incentives to file an injunction suit are concerned, no clear ranking can be established between the government agency and a private plaintiff. The answer to the question which party has more to gain from an injunction suit depends to a great deal on the shares of naïve and rational consumers in the population. Whereas the private plaintiff has higher incentives for filing an injunction suit for medium to large shares of naïve consumers, the opposite is true when the consumer population is mostly rational. In the latter case, the government agency obtains relatively large benefits from informing consumers also about a firm's high product quality.

Our results can be explained by two aspects: First, there are fundamental distortions due to differences in prices charged by the firms and due to fact that the naïve consumers do not critically assess unchallenged advertising messages. Second, there is a commitment problem for plaintiffs, as advertising decisions precede filing decisions. As a consequence, it can be optimal from a social welfare point of view to allow a more aggressive private plaintiff to file injunction suits instead of the government agency.

Our results shed light on the optimal law enforcement regime regarding unfair competition through false advertisements about product quality. The results indicate that private enforcement can outperform public enforcement, but this is dependent on the court system (affecting the size of trial costs) as well as on the characteristic of the consumer base. In our setting, private enforcement is more likely to outperform public enforcement for medium and high shares of naïve consumers. On the other hand, when there are consumer emancipation or education programs in place, our results indicate that it makes more sense to give a government agency the chance to take legal action against presumably false advertising claims. Hence, we can conclude from our analysis that from a public policy point of view, the decision for which party should be given the right of filing an injunction suit can directly be related to the share of naïve consumers as long as injunction suits come at reasonable costs. This finding is a direct consequence on how

the incentive to file an injunction suit depend on the degree of consumer naïveté. Note that in light of the experimental and empirical contributions on consumer naïveté (see the Introduction), it appears to be a realistic endeavor to evaluate the degree of consumer sophistication in the market and derive the optimal plaintiff regime from that.

Several further avenues for research could be considered. For example, the public good character of an injunction suit in the event of a larger set of competitors appears worthwhile to be addressed. This points to problems of free-riding that may call for other plaintiffs such as industry chambers, which are explicitly mentioned as potential plaintiffs in German law (*Act Against Unfair Competition (UWG)*, §8) for example. Furthermore, so far, we did not allow for errors in court decisions, which may raise the potential for using injunction suits as an instrument to deliberately deter market entry (as conjectured by Jordan and Rubin, 1979). In addition, in the event of repeated interactions between consumers and firms, reputation effects may also help to reduce the extent of false advertising.

8 References

Andreoni, J. (1991), Reasonable doubt and the optimal magnitude of fines: should the penalty fit the crime?, RAND Journal of Economics 22, 385–395.

Armstrong, M., Chen, Y. (2013), Discount pricing, CEPR Discussion Paper No. 9327.

Barigozzi, F., Garella, P.G., Peitz, M. (2009), With a little help from my enemy: Comparative advertising as a signal of quality, Journal of Economics & Management Strategy 18, 1071–1094.

Belleflamme, P., Peitz, M. (2010), Markets and Strategies, Cambridge: Cambridge University Press.

Corts, K.S. (2013), Prohibitions on false and unsubstantiated claims: Inducing the acquisition and revelation of information through competition policy, Journal of Law and Economics 56, 453–486.

Corts, K.S. (2014a), Finite optimal penalties for false advertising, Journal of Industrial Economics 62, 661–681.

Corts, K.S. (2014b), The social value of information on product quality, Economics Letters 122, 140–143.

Deng, Y., Staelin, R., Wang, W., Boulding, W. (2018), Consumer sophistication, word-of-mouth and "false" promotions, Journal of Economic Behavior & Organization 152, 98–123.

Dixit, A., Norman, V. (1978), Advertising and welfare, Bell Journal of Economics 9, 1–17.

Ellison, G. (2006), Bounded rationality in industrial organization, in: Blundell, R., Newey, W.K., Persson T. (eds.), Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress, Cambridge: Cambridge University Press.

Emons, W., Fluet, C. (2012), Non-comparative versus comparative advertising of quality, International Journal of Industrial Organization 30, 352–360.

European Commission (2012), Protecting businesses against misleading marketing practices and ensuring effective enforcement – Review of Directive 2006/114/EC concerning misleading and comparative advertising.

Gabaix, X., Laibson, D. (2006), Shrouded attributes, consumer myopia, and information suppression in competitive markets, Quarterly Journal of Economics 121, 505–540.

Glaeser, E.L., Ujhelyi, G. (2010), Regulating misinformation, Journal of Public Economics 94, 247–257.

Hattori, K., Higashida, K. (2012), Misleading advertising in duopoly, Canadian Journal of Economics 45, 1154–1187.

Hattori, K., Higashida, K. (2014), Misleading advertising and minimum quality standards, Information Economics and Policy 28, 1–14.

Hattori, K., Higashida, K. (2015), Who benefits from misleading advertising?, Economica 82, 613–643.

Heidhues, P., Kőszegi, B., Murooka, T. (2017), Inferior products and profitable deception, Review of Economic Studies 84, 323–356.

Hotelling, H. (1929), Stability in competition, Economic Journal 39, 41–57.

Jordan, E.R., Rubin, H. (1979), An economic analysis of the law of false advertising, Journal of Legal Studies 8, 527–553.

Kamenica, E., Gentzkow, M. (2011), Bayesian persuasion, American Economic Review 101, 2590–2615.

Kartik, N., Ottaviani, M. Squintani, F. (2007), Credulity, lies and costly talk, Journal of Economic Theory 134, 93–116.

Kartik, N. (2009), Strategic communication with lying costs, Review of Economic Studies 76, 1359–1395.

Katz, A. (1990), The effect of frivolous lawsuits on the settlement of litigation, International Review of Law and Economics 10, 3–27.

Lazear, E.P. (1995), Bait and switch, Journal of Political Economy 103, 813–830.

Matsumura, T., Sunada, T. (2013), Advertsing competition in a mixed-oligopoly, Economics Letters 119, 183–185.

McAfee, R.P., Mialon, H.M., Mialon, S.H. (2008), Private v. public antitrust enforcement: A strategic analysis, Journal of Public Economics 92, 1863–1875.

Milgrom, P., Roberts, J. (1986), Relying on the information of interested parties, RAND Journal of Economics 17, 18–32.

Mizuno, M., Hiroyuki, O. (1990), Does advertising mislead consumers to buy low-quality products?, International Journal of Industrial Organization 8, 545–558.

Nagler, M.G. (1993), Rather bait than switch—Deceptive advertising with bounded consumer rationality, Journal of Public Economics 51, 359–378.

Nelson, P. (1974), Advertising as information, Journal of Political Economy 82, 729–754.

Polinsky, A.M. (1980), Private versus public enforcement of fines, Journal of Legal Studies 9, 105–127.

Polinsky, A.M., Shavell, S. (2007), The theory of public enforcement of law, in: Polinsky, A.M., Shavell, S. (eds.), Handbook of Law and Economics, Amsterdam: Elsevier.

Piccolo, S., Tedeschi, P., Ursino, G. (2015), How limiting deceptive practices harms consumers, RAND Journal of Economics 46, 611–624.

Rauhut, H. (2017), Game theory, in: Bernasco, W., Elffers, H., van Gelder, J.-L. (eds.), The Oxford Handbook on Offender Decision Making, Oxford: Oxford University Press.

Rhodes, A., Wilson, C.M. (2018), False advertising, RAND Journal of Economics 49, 348–369.

Russo, J.E., Metcalf, B.L., Stephens, D. (1981), Identifying misleading advertising, Journal of Consumer Research 8, 119–131.

Schmalensee, R. (1978), A model of advertising and product quality, Journal of Political Economy 86, 485–503.

Schulte-Nölke, H. (ed.) (2017), Neue Wege zur Durchsetzung des Verbraucherrechts, Springer.

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Published in: European Journal of Political Economy, 58 (2019), pp. 245-261.

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Published in: European Economic Review, 111 (2019), pp.360-379.

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